Grade-Level Learning Expectations

A New Challenge for Elementary Mathematics Teachers

The No Child Left Behind (NCLB) legislation (2001) requires states to create challenging academic content standards in mathematics that specify what students are expected to know and be able to do. In response to NCLB, most states have developed specific grade-level learning expectations (GLEs) in mathematics for students in the elementary grades. As a result, teachers and school district administrators have been given additional responsibilities for interpreting the state-level GLEs and aligning them with their district mathematics curriculum materials and lessons. Further, many elementary teachers have served or will serve on textbook adoption and evaluation committees, which consider whether textbooks align with state or district GLEs. This process would be straightforward if textbooks were designed specifically for each individual state or if the GLEs for specific grade levels were the same across the states. Unfortunately, learning expectations vary from state to state, as is evident in state GLE documents (Reys

By Barbara J. Reys, Kathryn Chval, Shannon Dingman, Melissa McNaught, Troy P. Regis, and Junko Togashi

Barbara Reys, reysb@missouri.edu, Kathryn Chval, chvalkb@missouri.edu, Shannon Dingman, swd9fb@mizzou.edu, Melissa McNaught, mdmpnb@mizzou.edu, Troy Regis, tprb62@mizzou.edu, and Junko Togashi, nonfat-yogurt@sbcglobal.net, are members of the Mathematics Education Group, University of Missouri–Columbia, Columbia, MO 65211. The group contributes to the mission of the Center for the Study of Mathematics Curriculum: to gather and disseminate information about the role and influence of curriculum in U.S. schools.

2006). Further, creating textbooks for each state is not economically feasible or necessarily advisable. To produce a textbook series that can be used in many states, publishers tend to produce textbooks that include many more topics than any one state calls for at a single grade level, increasing the likelihood of large textbooks that treat many topics superficially (Reys and Reys 2006).

To illustrate the differences across state GLE documents, we analyzed the mathematics learning expectations at one grade level—grade 4—of the ten most populous states. We focused on grade 4 simply as an example; we suspect that findings are similar across other grades. The purpose of this article is to point out the similarities and differences in grade 4 GLEs across these ten states and discuss the implications for elementary school teachers, teacher leaders, curriculum coordinators, and curriculum developers.

Analyzing the Grade-Level Learning Expectations

We compiled and analyzed grade 4 GLEs from ten states that publish mathematics standards: California, Texas, New York, Florida, Ohio, Michigan, New Jersey, North Carolina, Georgia, and Virginia (the bibliography indicates the state documents we analyzed). The school population of these states represents approximately 50 percent of the U.S. school population. **Table 1** shows the number of grade 4 GLEs in each of the ten states according to the five content strands identified by the National Council of Teachers of Mathematics in *Principles and Standards for School Mathematics* (2000): Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. **Table 1** clearly indicates the variation across the states regarding the number of GLEs as well as the variation in their distribution across strands.

To determine how many different GLEs were represented in the set of ten state standards for grade 4, we reviewed each set of learning expectations. We noted and eliminated duplicates across states (e.g., learning expectations that focused on the same general outcome) and tracked how many states included each remaining GLE. For example, all ten states include a grade 4 GLE similar to the following one found in the Texas document:

> Use place value to read, write, compare, and order whole numbers through the millions place. (Texas Education Agency 1998)

As a result of the review, we identified 108 different grade 4 GLEs across the ten state documents (subsequently referred to as the "combined set" of grade 4 GLEs). **Table 2** shows the distribution of these 108 GLEs organized by mathematical strands and substrands.

Emphasis by mathematical strands

As might be expected, the largest number of GLEs from the combined set (45 of 108) are within the Number and Operations strand. These 45 expectations focus on whole-number concepts and place value; fraction and decimal concepts; meaning of operations; computation; rounding and estimating; and problem solving. We found the most consensus

across state documents in regard to computation. Indeed, this concept also received the greatest emphasis across the states surveyed. Nine of the ten states specify expectations related to adding, subtracting, multiplying, and dividing whole numbers; eight of the ten states emphasize adding and subtracting fractions; and all ten states specify expectations related to adding and subtracting decimals. Although computation was a common emphasis, GLEs across the states differed as to whether the focus of computation was exploring strategies, developing proficiency with a subset of numbers (e.g., multiplying two digits by two digits), or demonstrating fluency with a particular operation or algorithm.

Table 1

Number of Grade-Level Learning Expectations (GLEs) for Grade 4 per State by Content Strand

	Number and Operations	Geometry	Measure-ment	Algebra	Data Analysis and Probability	Total Number of GLEs
California	16	11	4	7	5	43
Texas	15	7	3	4	3	32
New York	27	8	10	5	6	56
Florida	31	11	17	10	20	89
Ohio	15	8	6	6	13	48
Michigan	37	5	11	0	3	56
New Jersey	21	10	8	6	11	56
North Carolina	14	3	2	3	4	26
Georgia	23	10	5	3	4	45
Virginia	17	8	11	2	3	41

Within the Geometry strand, the combined set includes 18 GLEs from the ten state documents, categorized as follows: coordinate systems; properties of lines, angles, and geometric figures; problem solving in geometry; and transformations and tessellations. The majority of the geometry expectations (10 of 18) in the combined set were isolated to one, two, or three states, underscoring the lack of consensus regarding the focus of geometry instruction in grade 4. For example, each of the following expectations is found in only one state GLE document:

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- Draw the points corresponding to linear relationships on graph paper.
- Investigate geometry in nature and art.
- Use coordinates to give or follow directions from one point to another on a map or grid.
- Understand length relationships of vertical and horizontal lines between coordinates.
- Interpret two-dimensional representations of three-dimensional objects.

Twenty GLEs in the combined set were identified within the Measurement strand. These include unit conversion; units and tools of measurement; perimeter, area, and volume; and problem solving. Eight of these 20 were found in only one state document, again demonstrating a lack of consensus across the states. For example, six states expect fourth graders to select appropriate units for measuring perimeter, area, and volume; four states expect them to use formulas to find the perimeter and the area of squares, rectangles, and more complex shapes; and one state expects them to measure the surface area of cubes and rectangular prisms.

The Algebra strand includes 11 GLEs gleaned from the ten state documents, representing the smallest percentage of the five mathematical strands. These 11 include a focus on patterns and relationships; expressions; and equations. Five or fewer state documents account for most of the expectations in the Algebra strand. Three states expect fourth graders to solve problems involving equalities and inequalities, and one state document includes this expectation:

> Understand that equals added to equals are equal and [that] equals multiplied by equals are equal.

Fourteen GLEs were identified in the Data Analysis and Probability strand and further categorized into four substrands: collecting and organizing data; analyzing and interpreting data; probability; and discrete mathematics. In grade 4, emphasis on probability is on designing experiments and listing possible outcomes of events. Five of the ten states expect fourth graders to identify or use measures of central tendency, while the other five states make no mention of measures of central tendency.

Unique and common learning expectations for grade 4

Across all five mathematical strands, most expectations were concentrated in only a few of the state documents. In fact, 48 of the combined set of 108 GLEs (44%) were found in three or fewer state documents, with 28 of the 108 (26%) unique to one state document. The unique expectations tended to be the following:

- Learning expectations that are very specific (e.g., "Find the smallest number of colors needed to color a map or graph.")
- Learning expectations found at earlier or later grade levels in other state documents (e.g., "Measure surface area of cubes and rectangular prisms.")
- Outliers—that is, learning expectations not found elsewhere at any grade level or in any state document (e.g., "Represent numbers in bases other than base ten, such as base five.")
- Learning expectations of greater specificity, therefore assumed as part of other expectations in other state documents (e.g., "Demonstrate a sense of the relative magnitude of numbers.")

Finally, we identified the most common GLEs—that is, those included in at least six of the ten state documents. Only 34 of the 108 expectations met this criterion (see **table 3**).

As noted in **table 3**, only four GLEs for grade 4 are included in all ten state documents:

- Read, write, compare, and order whole numbers.
- Read, write, compare, and order decimals.
- Add and subtract decimals.
- Solve applied problems involving multiplication and division of whole numbers.

To summarize, the ten state GLE documents we analyzed varied greatly in the number, content, and range of mathematical goals specified for grade 4. Overall, we found the intersection of learning expectations across the ten state documents quite small, while the union of these learning expectations was quite large and varied.

Table 2

Number of Grade 4 GLEs in Combined Set by Strand and Substrand

Strand and Substrand	Number of GLEs per Substrand	Number of GLEs per Strand
Number and Operations		45
Whole numbers and place value	8	
Fraction and decimal concepts	9	
Meaning of operations	3	
Computation	10	
Problem solving in number and operations	3	
Properties	6	
Rounding and estimating	6	
Geometry		18
Coordinate systems	4	
Line, angle, and figure properties	10	
Problem solving in geometry	2	
Tessellations and transformations	2	
Measurement		20
Unit conversion	4	
Tools and units	6	
Perimeter, area, and volume	7	
Problem solving in measurement	3	
Algebra		11
Patterns and relations	6	
Expressions and equations	5	
Data Analysis and Probability		14
Collect and organize data	4	
Analyze and interpret data	4	
Probability	3	
Discrete math	3	
TOTAL		108

Implications

Textbook authors and publishers pay close attention to state content standards, particularly those of

Table 3

Grade 4 GLEs Included in at Least Six of the Ten State Documents

Strand	Learning Expectation	Number of States
Number and Operations	Read, write, compare, and order whole numbers	10
	Read, write, compare, and order decimals	10
	Add and subtract decimals	10
	Solve applied problems involving multiplication and division with whole numbers	10
	Add, subtract, multiply, and divide whole numbers	9
	Construct, use, and explain procedures for pencil-and-paper, mental, and calculator computation	9
	Estimate whole-number, fraction, and decimal computations	9
	Identify, model, and compare fractions and mixed numbers	8
	Identify and generate equivalent forms of fractions and decimals	8
	Add and subtract fractions with like and unlike (common) denominators	8
	Round whole numbers	7
	Check reasonableness of computations	7
	Identify equivalent fractions (familiar denominators) and justify using models or number lines	7
	Know when an estimate is appropriate	6
	Understand inverse relationship of addition/subtraction and of multiplication/division	6
	Equate whole-number word name, standard form, and expanded form	6
Geometry	Use properties to describe, classify, compare, and model two- and three-dimensional shapes	9
	Understand and apply concepts of lines, angles, and circles	8
	Identify, describe, and model parallel, perpendicular, and intersecting lines	8
	Recognize, model, describe, and use transformations	8
	Identify, locate, and plot ordered pairs in the first quadrant	6
Measurement	Measure using common tools and select appropriate units of measure	7
	Convert from one unit of measure to a larger or smaller unit of measure	6
	Estimate and measure weight/mass	6
	Use models and counting procedures to investigate length, area, volume, and perimeter	6
	Select appropriate units for measuring perimeter, area, and volume	6
Algebra	Use symbols to represent quantities	6
	Evaluate relationships using variables	6
Data Analysis and Probability	Create appropriate graphs of data, as well as multiple graphs of the same data	9
	Interpret one- and two-variable statistics	8
	Compare multiple representations of data	8
	Create a plan for data collection	7
	Design experiments and list all possible outcomes	7
	Identify trends in data and justify generalizations	6

the populous textbook-adoption states of Florida, Texas, and California, for help in determining the content focus of each grade-level mathematics textbook. Addressing many or all of the learning expectations noted in the state GLE documents results in a textbook that is aligned with—and marketable in—multiple states. However, the limited overlap of learning expectations across state GLE documents, combined with the large number of unique learning expectations, is likely to result in textbooks that give shallow treatment to many topics. Imagine a fourth-grade mathematics textbook that includes 160 lessons addressing 108 GLEs. Curriculum coherence—the logical development of ideas across time—is sacrificed when the goal is to "cover" many topics.

As teachers, teacher leaders, and curriculum coordinators examine mathematics textbooks during a textbook adoption process, they will interpret the meaning and relative emphasis of each GLE in their state and district content standards as well as determine whether the textbooks under examination align with the expectations. Although the textbooks may include lessons that address each of the state GLEs, it is also important for decision makers to determine how many textbook lessons do not align with the state standards—that is, how many align with standards in other states or at other grade levels within their state. A further question to consider is, To what extent are the core learning goals developed in depth?

Given the nature and content of available textbooks as well as the variance in learning expectations across states, teachers will likely be using textbooks that do not align perfectly with their own state content standards. Therefore, they will need to selectively use textbooks or supplement them with other curriculum materials to meet their state's unique learning expectations. Such selective use and supplementation of the textbook raises concerns regarding curriculum coherence—that is, if textbook authors develop and sequence instruction to achieve particular goals, what is the effect of skipping sections or supplementing the textbook with other materials?

Although our analysis focused on grade 4 expectations, our findings suggest that elementary teachers at other grade levels will encounter similar issues when considering the alignment of GLEs with mathematics textbooks. The situation we describe here is not new, although we believe that the recent development of new grade-level content standards in response to NCLB requirements has exacerbated an already difficult situation. Thus, curriculum developers, textbook publishers, school administrators, and teachers are faced with new challenges to implementing the "coherent curriculum" described in *Principles and Standards for School Mathematics* (2000).

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