Pre-Algebra PoW Packet

Bartering for Bananas

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Welcome!

This packet contains a copy of the problem, the "answer check," our solutions, teaching suggestions, a problem-specific scoring rubric, and some samples of the student work we received in February 2005, when ***Bartering for Bananas*** first appeared. It is LibraryPoW #3344.

The original version of **Bartering for Bananas** was written to use with 7th grade students in the School District of Philadelphia. It was inspired by the content presented in the unit, Comparing Quantities (Section A: Compare and Exchange) from Encyclopædia Britannica's: *Math in Context* series. View the student pages here on Britannica's site:

http://mathincontext.eb.com/teacher/algebra2/18_cq_sb.pdf

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to "PoW Members" or use this URL to go to *prealgpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=527 [Log in using your PoW username/password.]

The Problem

In ***Bartering for Bananas***, students are asked to find how many bananas Byron can get with his five fish.

The text of the problem is included below. A print-friendly version is available from the "Print this Problem" link on the current PreAlgPoW problem page.

Bartering for Bananas

Byron lives where the people trade goods they produce for other things they need. He has some fish that he has caught, and he wants to trade them for as many bananas as he can. He asks around to find out what is being traded and finds out the following:

- Five fish are worth the same as two loaves of bread.
- Six oranges are worth the same as two melons.
- One loaf of bread is worth the same as one banana plus three oranges.
- Four loaves of bread are worth the same as fourteen oranges.

Question: How many bananas can Byron get with five fish?

Answer Check

Byron could get 14 bananas.

If your answer does **not** match ours,

- and you got 2 bananas and some oranges or melons, did you realize that you should continue trading until you only have bananas?
- did you think in terms of ratios?

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- did you remember that this is a "real-life" trading story and you probably wouldn't be able to cut whole fruit into pieces?
- have you clearly shown and explained the work you did?

- are you confident that you could solve another problem like this successfully?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?
- does this problem remind you of experiences you've had?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

The key concepts in this problem are ratio and proportion and logical reasoning. Good bartering skills aren't necessary! The three solutions methods are pretty much the same way, though they vary from using words to using variables. The key in each of them is figuring out that one orange is worth the same thing as two bananas.

Method 1: Logical Reasoning

We're given four pieces of information.

5 fish equal 2 breads
6 oranges equal 2 melons
1 bread equal 1 banana and 3 oranges
4 breads equal 14 oranges

Byron has 5 fish. He could trade this for 2 breads.

To figure out what 2 breads are worth, we can rewrite the third and fourth statements, multiplying the first by 2 and dividing the second by 2.

2 breads equal 2 bananas and 6 oranges
2 breads equal 7 oranges

Since both of the (2 bananas and 6 oranges) and (7 oranges) are equal to 2 breads, that means they would also equal each other. This will tell us how oranges and bananas are related, which will in turn help us turn bread into bananas!

2 bananas and 6 oranges equal 7 oranges
2 bananas equal 1 orange

Byron has 2 breads, and we know that's the same as 7 oranges. And 1 orange is the same as 2 bananas. So he must have 14 bananas.

Method 2: Algebraic Reasoning

After reading each of the clues, I realized that the trade for melons is pointless because melons are not included in any of the other trades. I used each of the other three clues and wrote these equations:

2 Breads = 5 Fish
1 Breads = 1 Banana + 3 Oranges
4 Breads = 14 Oranges

I converted all the equations in terms of four breads:

(Equation 1) 4 Breads = 10 Fish
(Equation 2) 4 Breads = 4 Bananas + 12 Oranges
(Equation 3) 4 Breads = 14 Oranges

I used Equation 3 and Equation 2 and got

14 Oranges = 4 Bananas + 12 Oranges

which means:

2 Oranges = 4 Bananas

I figured out that 4 Breads = 4 Bananas + (24 Bananas) So...

4 Breads = 28 Bananas

Finally I took (Equation 1) and put fish in terms of bananas

Therefore 10 Fish = 28 Bananas

Then I simplified to get

5 Fish = 14 Bananas

Method 3: Algebra

I wrote algebraic equations from the information using these variables:

f = fish, b = banana, r = bread, o = orange, m = melon

The statements given in the problem can be written:

5f = 2r
6o = 2m
r = b + 3o
4r = 14o

To answer the question in the problem, I knew that I had to find how many bananas was equal to 5f.

The second equation isn't important to the problem because melons aren't used in any of the other three equations.

I multiplied the third equation by 4, so I got

4r = 4b + 12o

Since 4r also equals 14o, then 4b + 12o = 14o.

I subtracted 12o from both sides of the equation and divided both sides by 2 so then I got

2b = o

which means one orange equals 2 bananas.

If Byron had 5 fish, he could trade that for 2 loaves of bread.

According to the third equation, he could trade 2 loaves of bread for 2 bananas and 6 oranges. Since 1 orange equals 2 bananas, his 6 oranges are worth 12 bananas, and then I added the 2 bananas to get 14 bananas.

Teaching Suggestions

To help students start thinking about this problem consider providing cut-outs of the fruit or manipulatives to represent the fruit to act out the bartering that Byron may have done to get the bananas.

When this problem was first presented in 2005, one common mistake that we noticed in students' solutions was to stop at 2 bananas and 6 oranges and not figure out how to convert those oranges into bananas. This is addressed in the Answer Check by pointing out to students that they should keep going until they have only bananas, but many will miss that (or read it and not doing anything about it). Feel free to reiterate it if you are presenting the problem in class and students are drafting their solutions on paper. If you are having students submit online, encourage them to not only view but also "read" the Answer Check.

Another common mistake, which we consider to be a lapse in Interpretation, is to use equivalencies that include fractions of the items. While it is true mathematically that 1 bread is equal to 3.5 oranges, within the context of the problem it won't work. It's hard to trade for a fraction of an item. Remind students to think of all the juice that would be dripping from that half of an orange! When a student reaches a conclusion like this, we hope they might review the context of the problem and maybe say, "Okay, so I can't use that, but I *can* use the fact that 2 breads is equal to 7 oranges."

Resist the urge to give direct instructions on a specific approach. Ask students to paraphrase the problem to check on their understanding before they begin working on it. The context of this problem lends itself to and "Act It Out" activity! Ask questions that help them understand the language of the problem, visualize it, and discover patterns. Good questions help students clarify their thinking and give you useful information as well.

The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy that works for them. You

can see from the various methods that we have thought to use for this problem that there are different ways to approach this problem. And, we may not have thought of them all!

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources:
http://mathforum.org/prealgpow/puzzles/supportpage.ehtml?puzzle=416

The Problem Solving and Communication Activity Series document for this problem contains ideas and activities to help students with this problem:
http://mathforum.org/pow/support/activityseries/prealgpow_psc.416.pdf

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Scoring Rubric

On the last page is the **problem-specific rubric,** to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on any problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Pre-Algebra PoWs. Please let me know if you have ideas for making them more useful.

~ Suzanne <suzanne@mathforum.org>

Sample Student Solutions

Focus on Interpretation

In the solutions below, we've focused on students' "interpretation" of the problem, meaning that the student has interpreted the problem correctly and attempts to solve all the parts. As noted in the rubric, interpreting this problem means to understand the idea of equivalencies and that Byron must end up with only bananas.

The sample student solutions included in this packet represent a broad range of both writing and problem solving skills. They also show a range of understandings, and we've tried to address each student's individual misunderstanding or weakness with comments that suggest what might be a good next step for that student.

Cort
age 13

Interpretation
Novice

2.5 bananas

I made a chart

Cort gives us little to go on to decide how he has interpreted the problem.

I might ask to see the chart if he were a student in my classroom and that might give me more clues into his thinking.

I also might ask what he traded to get 2.5 bananas!

Bob
age 12

Interpretation
Novice

Byron will get 2 banannas

He gets 2 banannas and 2 melons.

Bob, like Cort, hasn't given us too many clues about what he is thinking.

I might ask him to read the problem again and instead of trying to answer it right away, to list as many things as he noticed.

Asking students to "notice" and "wonder" as they read a problem can be very helpful. Before they know it they are understanding more and worrying less about immediately finding the answer.

Eric
age 13

Interpretation
Apprentice

I think Bryon can get 2 bananas with 5 fish.

The reason I think he can get 2 bananas is because 5 Fish equal 2 Bread and 1 Bread equals 1 Banana and 3 Oranges. Therefore 2 Breads which equal 5 Fish give you 2 Bananas and 6 Oranges.

Eric has used the information in the problem that provide the obvious equivalencies but seems to have not realized that Bryron wants to trade for "as many bananas as he can."

I might ask him if he can continue trading with his oranges until Byron only has bananas.

Hailey
age 12

Interpretation
Apprentice

5 fish equals 14 bananas.

first,I figured that if 4 loaves of bread equals 14 oranges, then 1 loaf would equal 3 1/2 oranges. And we know that 1 loaf equal 3 oranges and 1 banana so 1 banana equals 1/2 orange. So 1 loaf would equal 3 1/2 oranges which would equal 7 bananas. 5 fish is worth 2 loaves which would mean 2 loaves would equal 14 bananas.

Hailey used the fact that mathematically speaking, one bread is equal to three and a half oranges. While this is true, if the context of the problem is considered, it won't work.

I might suggest an "Act It Out" activity to help Hailey realize that she can use her initial thinking but just needs to go one step further to continue working only with whole numbers.

Al
age 13

Interpretation
Apprentice

Byron can get 14 bananas.

5f=2l
6o=2m
l=1b+3o
4l=14o
l=3.5o

5 fish is 2 loaves is 2 bananas and 6 oranges
1 banana is 1/2 an orange
5 fish is 2 bread loaves is 14 bananas.

Al also hasn't remembered that in a bartering context, we wouldn't cut fruit. Because he is using algebra to think about the problem, it seems even more likely that the context could be forgotten.

I might suggest that Al join Hailey to act out the problem!

Anderson
age 10

Interpretation
Practitioner

Byron can get 14 bananas with five fish.

First, I wrote algebraic equations from the information. I got 5f=2br, 6o=2m, br=ba+3o, and 4br=14o, when f is fish, br is bread, etc. I knew that i had to find 5f=?ba. Then when I looked at the equations, I found out that the second equation isn't important to the problem. I multiplied the third equation by 4, so I got 4br=4ba+12o. If 4br=4ba+12o, and 4br also equals 14o, then 4ba+12o=14o. Then I subtracted 12o from both sides of the equation and divided both sides by 2 so then I got 2ba=o which means one orange equals 2 bananas. Now, if Byron had 5 fish, he could trade that for 2 loaves of bread. According to the third equation, he could trade 2 loaves of bread for 2 bananas and 6 oranges. Since 1 orange equals 2 bananas, his 6 oranges are worth 12 bananas, and then I added the 2 bananas to get 14 bananas.

Anderson has done a nice job of setting up equations, working with them to find equivalencies and making sure to stay with whole numbers!

Using "br" and "ba" indicate that he realizes that you can't use "b" both for bread and also bananas. I might point out that in algebra we usually only use one letter and he might choose "r" to represent bread and "b" for banana.

I might also suggest some ideas to format his explanation (some spacing and paragraph breaks. Revising the format would help readers view his solution and would improve his clarity score.

Daniel
age 12

Interpretation
Practitioner

Byron can get fourteen bananas with five fish.

From the given facts, 1 Loaf = 1 Banana + 3 Oranges. Also, 4 Loaves = 14 Oranges. We want to find out how many Loaves are equal to how many bananas. To cancel out Oranges, we have to find the least common multiple of 3 and 14. That number is 42. Multiply the first equation by 14 and the second equation by 3. Then subtract the second equation from the first equation. I found that 2 loaves equals 14 bananas. From the first given fact, five fish equals 2 loaves. Therefore, five fish also equals fourteen bananas.

Some students, like Daniel, have interpreted the problem well but can be encouraged to work on completeness.

Although a reader, familiar with algebra (in particular, solving systems of equations) can read through the lines, a pre-algebra student might not be able to "see" enough to know how all of this works out.

I would encourage Daniel to include more of his steps including how the equations looked as he multiplied by 3 and 14.

Angelle
age 13

Interpretation
Practitioner

Bryon can receive 14 bananas for 5 fish.

I first looked at all my clues as to how barter was done. Noticing that 5 fish equals the same as 2 loaves of bread I decided to use bread instead of fish for finding out my solution because it is used in more clues. Also, during this portion of my problem solving I found that 6 oranges are worth the same as 2 melons was not needed to find the answer to this problem.

First I notice that 1 loaf of bread is worth the same as 1 banana and 3 oranges. I found that since we are using 2 loaves of bread that we need 2 bananas and 6 oranges to replace 2 loaves of bread. I also found that 4 loaves of bread is worth as much as 14 oranges and since we are using only 2 loaves of bread I notice that 2 loaves of bread is the same as 7 oranges. Since you can get either 6 oranges and 2 bananas or 7 oranges for 2 loaves of bread I found that in replace for 1 orange you can receive 2 bananas. So instead of receiving 7 oranges for 2 loaves of bread you can receive 14 bananas for 2 loaves of bread to replace each orange.

1 bread = 1 banana + 3 oranges so...
2 bread = 2 bananas + 6 oranges and also we know...
4 bread = 14 oranges so...
2 bread = 7 oranges
1 orange = 2 bananas so...
2 bread = 14 bananas so if...
5 fish = 2 bread then...
5 fish = 14 bananas

Therefore since 5 fish equals 2 loaves of bread and 2 loaves of bread equal 14 bananas then 5 fish equal 14 bananas.

Reflection: I enjoyed it because it seemed more like a logic type problem than a math problem.

Angelle has done a nice job, explaining her what she was thinking as she read the problem. Besides interpreting the problem well, she has done a very clear and complete job with her explanation.

Pre-Algebra Scoring Rubric for *Bartering for Bananas*

For each category, choose the level that *best describes* the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	does none or one of the things listed under Practitioner	does two or three of the things listed under Practitioner	attempts to find the number of bananas Byron can get for his 5 fish understands the idea of equivalencies understands that Byron must end up with only bananas understands that the equivalencies can be increased or reduced proportionally, except that the items must remain whole – you can't trade for half an orange, etc.	there is no Extra, and no way to be an Expert in this category for this problem
Strategy	does not have any ideas about how to solve the problem	has some ideas about how to solve the problem, but isn't quite there	has a strategy that relies on skill, not luck uses correct equivalencies (though "may" use fractional parts in their strategy – that makes them an Apprentice in Interpretation)	uses algebra, as in variables and equations (as opposed to just algebraic reasoning)
Accuracy	has made many errors	makes a few errors that lead to an incorrect answer	work is accurate and contains no arithmetic mistakes uses appropriate language and units, such as breads of fruits	[not normally available for this category]
Communication				
Completeness	has written nothing that tells you how they found their answer	shows work without an explanation *or* explains everything without showing the numbers doesn't include enough information for another student to follow	attempts to explain all of the steps taken to solve the problem, which might include • any trades made and why they're legal • any calculations with accompanying explanation • where any equations came from	adds in useful extensions and further explanation of some ideas involved
Clarity	explanation is very difficult to read and follow	another student wouldn't be able to follow their explanation entirely long and written in one paragraph lots of spelling errors/typos	explains all of the steps mentioned in such a way that another student would understand makes an effort to check their formatting, spelling, and typing (a few errors are fine)	formats things exceptionally clearly answer is very readable and appealing
Reflection	*The items in the columns to the right are considered reflective, and could be in the solution or the comment they leave after viewing our answer:*	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used	comments on and explains the ease or difficulty of the problem **revises their answer and improves anything**
	does nothing reflective	does one reflective thing	does two reflective things	does three or more reflective things or an great job with two