



Geometry PoW Packet

Flying High

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Welcome!

This packet contains a copy of the problem, the “answer check,” our solutions, teaching suggestions, and a problem-specific scoring rubric. Since it’s a new problem, there are not any student solutions.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to “PoW Members” or use this URL to go to *geopow-teachers* directly: <http://mathforum.org/kb/forum.jspa?forumID=529> [Log in using your PoW username/password.]

The Problem

In *Flying High*, students try to figure out which of two airplanes is gaining altitude more quickly. A key to the problem is picking a good representation for the situation.

The text of the problem is included below. A print-friendly version is available from the “Print this Problem” link on the current GeoPoW problem page.

Flying High

Which gains altitude more quickly, a plane traveling 400 mph and rising at an angle of 30 degrees, or a plane traveling 300 mph and rising at an angle of 40 degrees? How much more quickly (in mph) does it gain altitude?

Extra: Change one angle or one speed so that the two planes are rising at the same rate.



Answer Check

The first plane is rising about 7 mph faster than the second plane.

If your answer **does not** match our answer,

- did you try modeling the problem with two right triangles?
- did you realize that the given speeds are air speeds, not ground speeds?
- and you did use right triangles, did you realize that the bases of the two triangles would not be equal?

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you’re still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- and you used right triangles, did you say something about how you set up your triangles?
- are there any hints you would give another student?
- what was the hardest part of solving the problem?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

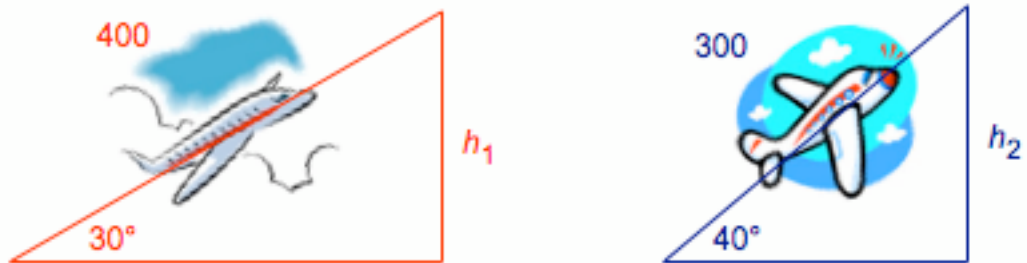
Our Solutions

The key concept in this problem is right triangle trigonometry.

We have two planes, one traveling 400 mph and rising at a 30 degree angle, and one traveling 300 mph and rising at a 40 degree angle. We’re asked to figure out which one is gaining altitude more quickly, and how much more quickly.

We can model the problem using right triangles. If we consider what happens in one hour, the first plane travels 400 miles and the second travels 300. Those would be the lengths of the hypotenuse of

each triangle. Using the angle at which they're rising, we can find the height of each triangle. That will represent how much altitude each plane gains in one hour, so we have altitude in miles per hour. It will look something like this:



We want to know the side opposite the angle, and we know the hypotenuse. This calls for the sine ratio, which is opposite/hypotenuse.

$$\sin 30 = \frac{h_1}{400}$$

$$400 \sin 30 = h_1$$

$$200 = h_1$$

The first plane is rising at 200 mph. Now let's do the second plane.

$$\sin 40 = \frac{h_2}{300}$$

$$300 \sin 40 = h_2$$

$$192.8 = h_2$$

The second plane is rising at about 193 miles per hour. So the first plane is rising about 7 mph faster.

Extra: We're asked to figure out how to make the planes climb at the same rate by changing either a speed or an angle. That means making h_1 and h_2 equal. I'll first try it by changing the angle at which the second plane is rising. I'll call it a .

$$400 \sin 30 = 300 \sin(a)$$

$$\frac{400 \sin 30}{300} = \sin(a)$$

$$4/6 = \sin(a)$$

$$41.8 = a$$

So plane 2 needs to rise at a slightly steeper angle of 41.8 degrees.

I'm curious how much the angle of the other plane would have to drop—is it a similar amount? Let's find out.

$$400 \sin(b) = 300 \sin 40$$

$$\sin(b) = \frac{300 \sin 40}{400}$$

$$\sin(b) = 0.482$$

$$b = 28.8$$

So that drops a little bit more than the other one rose. I could do the same exploration with the speeds, but will leave that to the reader!

We could use the same setup to find a new value for either speed or either angle.

Teaching Suggestions

The most interesting part of this problem is deciding how to represent it. Since the given information is a rate and an angle, the idea of using a triangle (which has angles, but lengths instead of rates) might not occur to students at first. Have them draw some pictures of what's happening – not necessarily triangles, but students will probably draw a line going up at an angle, at the very least. Then you might

ask them to think about how they might find the height of the plane at any given time, or at least after 1 hour, 2 hours, 3 hours, etc.

Some students might set up a triangle but then assume that the bottom of the triangle represents 1 hour in each case. It's a natural extension from a time/distance graph, where the horizontal axis is often time, but doesn't work in this case. We can set up a triangle that represents what happens over the course of one hour, but that hour isn't an actual part of the triangle.

There might be some confusion about whether the speeds of the planes represent airspeed or ground speed. You might let some students reason that out – which makes more sense? Could we solve it either way? Or you might tell them that when ground speed is given, it's usually labeled as such. Since here we just have speed, we can assume it's airspeed.

Students might also do something like $40 * 300 = 30 * 400$, and conclude that they both must be rising at the same rate, since those are equal. That is related to saying that in a triangle, the side across from the 30 degree angle must be twice as long as the one across from the 15 degree angle. But we know that's not the case. In fact, sides are proportional to the sines of the angles, not the angles themselves. To help them think about this more, you might give them the example of a 30-60-90 triangle, with which they are probably familiar.

(The day we talked about this problem at the Math Forum office, I happened to tutor a kid who was doing the Law of Sines and the Law of Cosines. We had a 30-60-90 triangle, and he had just finished putting the side lengths when he said, pointing at the diagram, "Wait! That angle is twice that angle, so shouldn't that side be twice that side?" He had seen that diagram many times before, but had just noticed that it didn't work the way he would have expected it to. We had a great talk about it, looking at a number of familiar examples, and trying to draw non-equilateral triangles that had that proportional relationship between the sides and angles. Needless to say, we were quite unsuccessful!)

You can find more online resources to go along with this problem at

<http://mathforum.org/geopow/puzzles/supportpage.ehtml?puzzle=417>

All the resources to go with this and all other Current Geometry PoWs from this year are linked from

<http://mathforum.org/pow/support/>

Scoring Rubric

On the last page is the **problem-specific rubric**, to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on any problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Geometry PoWs. Please let me know if you have ideas for making them more useful.

~ Annie

Geometry PoW Scoring Rubric for Flying High

For each category, choose the level that *best describes* the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	does one or none of the things listed under Practitioner	does only two of the things listed under Practitioner	understands that the situation can be modeled with a right triangle and that the angle given is an angle of inclination attempts to figure out which plane is rising more quickly attempts to find how much more quickly	is at least a Practitioner in Strategy and has solved the Extra question
Strategy	has no ideas that will lead them toward a successful solution	has a strategy that relies on luck might construct and measure a drawing might attempt to solve the problem based directly on the angle measures rather than the sines (or cosines, or tangents) of the angle measures	has a strategy that relies on sound reasoning, not luck might use the sine function to find the height of the triangle	solves the problem using parametric equations
Accuracy	has made many errors	has made several mistakes, or has used vocabulary or units incorrectly	makes no mistakes of consequence and uses largely correct vocabulary and notation, including any variables used	[generally not possible]
Communication				
Completeness	has written almost nothing that tells you how they found their answer	shows work without explanation, or gives an explanation without showing any work	shows and explains the steps taken and why they are reasonable steps, which might include: <ul style="list-style-type: none"> • how any triangles were set up • what formulas or functions were used and why they were the right choice 	includes additional helpful information, doesn't just add more for the sake of adding more
Clarity	explanation lacks clarity and organization	explanation is difficult to follow length warrants separation into more paragraphs lots of spelling errors/typos	explains the steps that they <i>do</i> explain so that another student would understand (needn't be complete to be clear) makes an effort to check their formatting, spelling, and typing (a few errors are okay)	answer is clearly written and well-organized formats things very clearly
Reflection	<i>The items to the right are considered reflective, and could be in the solution or their comment:</i> does nothing reflective	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer does one reflective thing	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used does two reflective things	comments on and explains the ease or difficulty of the problem revising their answer and improving anything does three reflective things or great job with two