Math Fundamentals PoW Packet

Sports Weigh In

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Welcome!

This packet contains a copy of the problem, the "answer check," our solutions, teaching suggestions, and a problem-specific scoring rubric. ***Sports Weigh In*** has been chosen from the Problem Library (#3643), so I'm also including some sample student solutions.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to "PoW Members" or use this URL to go to *funpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=526 [Log in using your PoW username/password.]

The Problem

Sports Weigh In involves logical reasoning to determine how much each kind of ball weighs. The arithmetic requirements are minimal: doubling, halving, and subtracting of relatively small numbers.

The Extra asks solvers to find all the other ways of combining these balls to total 22 ounces and also to show either explicitly or indirectly through their strategy, that they have found all possible solutions.

A print-friendly version is available from the "Print this Problem" link on the problem page.

Sports Weigh In

The Math Club at Dawn's school had a Math Carnival. At her booth Dawn set up three scales with baseballs, tennis balls and soccer balls, as shown in the picture below. Contestants tried to determine the weight of each kind of ball.

How much does each kind of ball weigh?

Explain how you solved it. Show how you know you are correct.

[*Assume that all the soccer balls weigh the same, all the baseballs weigh the same, and all the tennis balls weigh the same.*]

Extra: Find all the other combinations of these kinds of balls that would weigh 22 ounces. Explain how you found them and how you know you have found them all.

Answer Check

The baseballs weigh 5 ounces each. Now you can find the rest.

If your answer doesn't agree with ours —

- can you find how much one soccer ball and one tennis ball weigh together?
- did you try doubling the scale on the left? Can you compare that to another scale?
- did you make estimates and test them? Make a list to help you keep track of the numbers you test and to make a better next guess.
- did you check your arithmetic?

If you used guess-and-test, did you tell . . .

- what numbers you tried and how you tested them?
- how you knew whether they worked or not?
- how you decided what to try next?

If your answer does agree with ours —

- is your explanation clear and complete?
- did you try the Extra question?
- did you verify your answer with another method?
- did you show how you know your answer is right?
- did you have any "Aha!" moments? Describe them.

Our Solutions

Below are several examples of ways I imagine children might solve the problem. These solutions are not meant to be prescriptive or comprehensive. I hope you will share any other approaches you or your students use on the *funpow-teachers* discussion board, along with any teaching strategies you found to be successful.

Strategy 1 — Guess-and-test

[*While I expect many students to use this strategy, guess-and-test is not as well suited to this problem as it is to many others. It is not simple to gather information from incorrect guesses and use it to make adjustments*.]

I used guess and test. I compared the first and last scales and saw that a baseball must weigh 3 oz more than a tennis ball. I tried three weights that would maintain that difference and work on the first scale. Then I found out what the others would be. I made a table of how that would work. When I adjusted after each test, I kept the total of all three balls at 19 oz.

	B + S + T = 19 oz	2S + 2T = 28 oz	S + 2B = 22 oz	
B=7 S=8 T=4	7 + 8 + 4 = 19 oz	16 + 8 = 24 oz	8 + 14 = 22 oz	Need more weight on middle scale without changing totals of first and third. Try heavier soccer ball, must be even number.
B=6 S=10 T=3	6 + 10 + 3 = 19 oz	20 + 6 = 26 oz	10 + 12 = 22 oz	Getting closer. Make soccer ball heavier.
B=5 S=12 T=2	5 + 12 + 2 = 19 oz	24 + 4 = 28 oz	12 + 10 = 22 oz	Yes!

The tennis ball weighs 2 oz, the soccer ball weighs 12 oz, and the baseball weights 5 oz. The bottom row of my table shows that those weights are correct.

Strategy 2 – Logical reasoning, halving the middle scale, with Extra:

Since the middle scale contains 2 soccer balls and 2 tennis balls, one soccer ball and one tennis ball together must weigh half of that, or 14 oz. Comparing that to the 19 oz scale tells me that the baseball weighs 5 oz. Total - (soccer + tennis) = 19 - 14 = 5 oz.

Looking at the third scale, I can tell that the soccer ball must weigh 12 oz.
Total - 2 baseballs = 22 - 10 = 12 oz.

Looking back at the first scale, I can tell that the tennis ball must weigh 2 oz.
Total - (soccer + baseball) = 19 - 17 = 2 oz.

I tested my values to make sure they worked on the scales. T stands for the weight of a tennis ball, S for a soccer ball, and B for a baseball.
T + S + B = 2 + 12 + 5 = 19 oz.
2S + 2T = (2 * 12) + (2 * 2) = 24 + 4 = 28 oz.
S + 2B = 12 + (2 * 5) = 12 + 10 = 22 oz.

Extra: I made a table to find all the other ways to make 22 oz. I started with the greatest number of soccer balls possible, then the maximum number of baseballs, and decreased systematically. I knew that any combination including baseballs must use an even number of them, because 22 is even.

Soccer @ 12 oz	Baseballs @ 5 oz	Tennis @ 2 oz	Total weight
1		5	12 + 10 = 22 oz
	4	1	20 + 2 = 22 oz
	2	6	10 + 12 = 22 oz
		11	11 * 2 = 22 oz

Strategy 3 – Logical reasoning, doubling the first scale:

I doubled the first scale. 2 baseballs + 2 soccer balls + 2 tennis balls = 38 oz. I compared that to the middle scale. The difference of 10 oz (38 - 28 = 10) must be due to the 2 baseballs, so one baseball weighs half that, or 5 oz.

I subtracted the weight of 2 baseballs from the third scale to find the weight of a soccer ball. 22 - 10 = 12 oz.

When I subtracted the weight of a soccer ball and a baseball from the first scale, I found that a tennis ball weighs 2 oz. 19 - 12 - 5 = 2 oz.

I tested my values to make sure they worked ... [*as in Strategy 2*]

Strategy 4 – Algebraic:

I let S stand for the weight of a soccer ball, B for that of a baseball, and T for that of a tennis ball. I used the information from two scales to express T and B in terms of S.

From the middle scale: 2S + 2T = 28
Divide both sides by 2: S + T = 14
T = 14 - S

From the scale on the right: S + 2B = 22
Divide both sides by 2: S/2 + B = 11
B = 11 - S/2

Substitute for T and B into the first scale: S + T + B = 19
S + (14 - S) + (11 - S/2) = 19
Collect like terms: 14 + 11 - 19 = S - S + S/2
S/2 = 6
S = 12

Substitute for S on the middle scale: 24 + 2T = 28
2T = 4
T = 2

Substitute for S on the right: 12 + 2B = 22
2B = 10
B = 5

I tested my values for S, T and B on the 3 scales to check my answer:
5 + 12 + 2 = 19 oz
24 + 4 = 28 oz
10 + 12 = 22 oz

Teaching Suggestions

Sports Weigh In requires understanding that all soccer balls are of equal weight, as are all baseballs and all tennis balls. The goal of this problem is to develop logical reasoning, not simply arrive at an answer.

As mentioned in Strategy 1 above, guess-and-test is not the most effective way to solve the problem, but many students are confident of that approach, and will try it. You might encourage them to find an alternative approach, at least to verify their answers. Ask them to look carefully at the scales and try to notice relationships that can lead to a more direct and insightful solution. The questions in the answer check above can be useful in helping students make such discoveries.

Students who do approach the problem with a guess-and-test strategy (Strategy 1 above) need to make use of the information gained from incorrect attempts in order to make a better next guess – achieving success through skill and understanding, not pure luck. Some students might uncover ways that parity (odd/even) can help them. See the answer check, above, for more details about effective use of guess-and-test.

The Extra provides an opportunity for students to find a systematic method of ensuring they have found all combinations. The samples below display several ways to do that.

The Teacher Support Page for this problem contains links to related problems in the Problem Library and to other web-based resources: http://mathforum.org/funpow/puzzles/supportpage.ehtml?puzzle=417

Scoring Rubric

On the last page you will find the **problem-specific rubric,** to help those who are assessing student solutions. It specifies what we expect from students in three areas of problem solving and three areas of communication. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on every problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

Sample Student Solutions

Focus on Strategy

I've chosen the samples below to illustrate a range of **Strategies** demonstrated by submitters. This category of our rubric addresses whether the solver has applied an approach that is systematic and involves sound mathematical reasoning. It is not my purpose here to give a definitive judgment, but rather to highlight the range and variety of work done by students and to suggest ways I might help them to take next steps.

You can view commentary and solutions that were highlighted in 2006 at this Library page:
http://mathforum.org/librarypow/office/teachers/index.ehtml?puzzle=3643&page=lib_solution

Kate
Age 13

Strategy: **Novice**

One soccer ball is 12oz, one baseball is 5oz, and one tennis ball is 2oz.

On the first scale the number of ounces is 19, and there is one of each ball.

If you look at the size of the balls, the soccer ball is twice as big as the baseball, and the tennis ball is also smaller than the baseball.

All three balls equal 19oz.

Step 1: 6x3=18 each ball is now 6 oz

Step 2: The soccer ball is twice as big than the other balls. Now the soccer ball is 12oz and the other to balls are 3oz.

Step 3: And remember the tennis ball is even smaller than the baseball, so the numbers change agian. The tennis ball is now 2 oz and the baseball is now 4os.

Step 4: Finally, remember in step i we did 6x3=18, well there's one more number to add. The tennis ball stays the same at 2oz, but the baseball becomes 5oz

Step 5: 12oz+5oz+2oz=19oz
All three balls equal 19oz.

Kate's first step of using the mean of the weights as a starting number is reasonable, but her strategy fails after that. I wonder if she heard the answer from a peer and invented a strategy to explain it. It seems to be based loosely on the relative sizes of the balls, but with no sound rationale that would account for the specific numbers. There is no reference to the other two scales. For a 13-yr old, this indicates a Novice in Strategy. Can she explain why the solution cannot be another combination that totals 19, such as 11, 5 and 3? We'd use her reasoning from that to move down a more logical path.

Samantha
Age 9

Strategy: **Apprentice**

The soccer ball weighs 12 ounces, the baseball weighs 5 ounces, and the tennis ball weighs 2 ounces.

Extra: 1 soccer ball and 2 baseballs; 1 soccerball and 5 tennis balls; 2 baseballs and 6 tennis balls; 4 baseballs and 1 tennis balls; and 11 tennis balls.

I used different combinations until I came up with the correct numbers. I know I am correct because I was able to use these weights for all three problems and I got the amount asked for (e.x. 12+5+2=19, 12+12+2+2=28, 12+5+5=22).

Samantha verified her solution by showing the sums of the weights on each scale using her solution. We can't tell whether she applied her guess-and-test approach systematically or randomly or how she made decisions as she tested. I'd ask for an example of a trial that didn't work. She did make insightful observations in the Extra.

Extra: Since a soccer ball weighs 12 ounces, I could only us it once in a combination. I started from that point to come up with my combinations. A baseball weighs 5 ounces, so I had to multiple it by an even number. Thats's why I only used 2 in one combination and 4 in the other. Lastly, I used the tennis ball combination by dividing 22 by 2 and making my combinations with the tennis ball.

Lise
age 8

Strategy: **Practitioner**

My answers are that the soccerball weighs 12 oz.,the tennisball is 2 oz.,and the baseball is 5 oz.

First I added 13 + 13 = 26 to try to find out what the soccerball weighed.I was looking at the middle weight.Then I saw that the tennisball weighed 1 oz.if I did it that way. I thought that wouldn't be enough oz.So next I tried 12 oz.12 + 12 = 24.That would leave me 4 oz.That would be enough for 2 tennisballs because each would be 2 oz.To check it I tried it in the first weight and the 3rd weight.First I tried it in the first weight. I saw a soccerball,a baseball,and a tennisball.I tried adding my answers for the soccerball and tennisball.2 + 12 =14 oz.I was trying to get up to 19 oz.so then 19 - 14 = 5 oz.Then the baseball would equal 5 oz.Then to check my answers again I went to the third weight and saw 1 soccerball and 2 baseballs.12 oz (soccerball)+5 oz.(baseball)+ 5 oz. (another baseball)= 22 oz. wich is the total weight that I'm trying to get to.That is how I got my answers and checked my answers.

Lise described a systematic guess-and-test, giving reasons for choosing a starting number and adjusting it. She told how she tested her guess and how she knew it was right. One space between sentences would improve readability. I think she's ready to try the Extra.

Mumbi
age 10

Strategy:
Practitioner

The tennis balls weigh 2oz. The Soccer ball weighs 12oz. Baseballs weigh 5oz.

To find my answer I knew that 2 soccer balls and 2 tennis balls were 28oz. So, to the weight of 1 soccer ball and 1 tennis ball, you would have to divide 28 by 2 getting you 14oz.

I also knew that 1 baseball, 1 soccer ball and 1 tennis ball weighed 19oz. Knowing that 14oz were taken up by the soccer ball and tennis ball, what ever was left must have been the weight of the baseball. So, I subtracted 14 from 19 and got 5oz as the weight of the base ball. So if the balls weighed 14oz. togethor plus 5 that would be 19oz.

Mumbi's articulate explanation of logical reasoning shows clear thinking and an ability to see the relationships among the balls and weights. I would ask to see the calculations that verify his solution. It would be interesting to see how he would apply his reasoning ability to solve the Extra.

Also knowing that 2 baseballs and 1 soccer ball was equal to the weight of 22oz, and knowing that each baseball was 5oz, 2 of them would equal 10oz, and the rest would be the weight of the soccer ball. So then I subtracted 10oz from 22oz, and got 12oz as the weight of the soccer ball.

Then, if the soccer ball was 12oz and the baseball was 5oz and along with the tennis ball, the whole weight was 19oz, I knew that subtracting 17oz. from 19oz. Leaving 2 ounces for the weight of the soccer ball. So 12oz. plus 5oz. plus 2oz. Is 19oz.

Becky
age 9

Strategy:
Practitioner

Each soccer ball weighs 12oz, each baseball weighs 5oz, and each tennis ball weighs 2oz.

First I knew i had to figure out the number of oz each ball weighed so here were my steps: First I looked at the two end scales. They are the same exept for the one scale has a tennis ball instead of a baseball.The scale with the tennis ball was 3oz less than the scale without the baseball.That told me the tennis ball weighed 3oz less than the baseball. I pretended the baseball was a tennis ball .Then I knew that 3oz less than 19oz would be 16oz.then I looked at the scale in the middle and saw that scale had one more soccer ball than the first scale.I did 28oz minus 16oz and got 12oz ,so I knew the soccer ball weighed 12oz.Then once again I looked at the end scale and saw I needed two equall numbers to equall ten so I could 22oz.Of course the number was 5,so I put a 5 above every baseball.I looked back over at the first scale and saw there was only one number missing witch was 2.Each soccer ball weighs 12oz,each baseball weighs 5oz and each tennis ball weighs 2oz.

Becky applied logical reasoning in a very creative way. She compared the first and third scales to find the difference between the baseball and the tennis ball. I'd model ways to use more number models to improve Clarity and ask her to verify her solution by showing that her

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Archit
age 11

Strategy:
Expert

THe soccer ball is 12 oz, the baseball 5 oz, and the tennis is 2 oz.

To find it, suppose s is the weight of the soccer ball, t is the weight of the tennis ball, and b is the weight of the baseball. The following are given:
I. b+s+t=19 II. 2s+2t=28 III. 2b+s=22

II can be reduced to s+t = 14. That can be fitted into b+s+t=19, making it b+14=19. That solves to b=5. From that, fit the value of b into III, so 2(5) + s = 22 or s = 12. Last, fit s=12 into s+t=14. That results in t=2.

Archit's algebraic approach earns Expert in Strategy. He provides clear rationale to support his finding all 22 oz Extra combinations.

EXTRA: First, there can only be 1 or 0 soccer balls, because 2 soccer balls weigh 24 oz which exceeds 22 oz. With 1 soccer ball, the combination of tennis and baseballs must be 10. Therefore, there can be 5 tennis balls or 2 baseballs. With 0 soccer balls, the sum of the baseballs and tennis balls must be 22. There can not be more than 4 baseballs, since 5 baseballs weigh 25, which is more than 22. Also, the number of baseballs must be even, since otherwise we can not make up the difference with tennis balls. So, there are only three possibilities, 4 baseballs, 2 baseballs or no baseballs. With 4 baseballs there is one tennis ball needed. With 2 baseballs there are 6 tennis balls and with 0 baseballs there are 11 tennis balls. So, there are 5 possible combinations and no others: [*Correct answers omitted to save space*.]

Ann
age 10

Strategy:
Expert

Extra Explanation: There are a maximum of 7 possibilities. They are:
(1) has all three kinds of balls
(2) has only soccer and base balls
(3) has only soccer and tennis balls
(4) has only base and tennis balls
(5) has only soccer balls
(6) has only base balls
(7) has only tennis balls.

In order to make the balls equal to 22 oz., there is no answer to (1), there is one answer to (2), 1 answer to (3), two answers to (4), no answer to (5) and (6), and one answer to (7).

Ann's correct solutions to the main problem and the Extra have been omitted to save space. Her main problem strategy mirrors Mumbi's. Her Extra approach is very analytical and systematic, earning Expert status.

Please let me know if you have ideas for making these packets more useful.

~ Claire claire@mathforum.org

The Math Fundamentals Problem of the Week Scoring Rubric — **Sports Weigh In (posted 20 April 2009)**

For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands that all soccer balls are of equal weight, as are all baseballs and all tennis balls. Understands that each kind of ball is a different weight. Makes a reasonable attempt to find the weights of the three kinds of balls.	Solves the main problem correctly. Achieves at least Practitioner in Strategy. Understands the Extra; seeks all other combinations of balls weighing 22 oz.
Strategy *(NB: based on their interpretation of the problem)*	Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that makes sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically. OR Strategy is not evident. Might verify a correct answer, but fail to explain how they found it.	Approaches problem systematically. Applies a sound strategy, achieving success through skill, not luck. Any guess-and-test approach must reflect reasoned decisions. Chosen strategy accounts for any answer(s) that changed after checking our answers.	Does **one or more** of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology (spreadsheet) or algebra.
Accuracy *(NB: based on chosen strategy)*	Has made many errors. OR Shows/describes no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic errors.	Not available for this problem.
Communication				
Completeness *(NB: an incorrect solution can be complete)*	Writes very little to explain how the answer was achieved.	Provides explanation but does not include calculations; OR Shows calculations without explanation about why they were done.	Explains most of the steps taken to solve the problem and the rationale for them, with enough detail for another student to understand. Includes numbers and key calculations. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Does **one or more** of these: Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight. Demonstrates that the answer works. Includes a relevant table of data.
Clarity *(NB: incomplete and incorrect solutions can be explained clearly*	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Uses level-appropriate math language and correct units: ounces or oz. Shows effort to use good formatting, organization, spelling, grammar, typing. Errors don't interfere with readability.	Formatting and organization make ideas exceptionally clear. Answer is very readable and appealing, might include a helpful diagram or image. (A diagram or image alone doesn't qualify for Expert status.)
Reflection (See list below.)	Does nothing reflective.	Includes one reflective thing.	Includes two reflective things.	Includes 3 or more reflective things or does an exceptional job with 2 of them.
	The items to the right are considered reflective, and could be in the solution OR in the comment a student leaves after viewing our answer:	• **Revises and improves the submission**. • Checks the answer using a different method. • Explains a hint she/he would give someone.	• Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them. • States any assumptions made in the solving process.	• Describes something learned from the problem. • Comments on AND explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used. • Describes any "Aha" moments.