Math Fundamentals PoW Packet

She Counts Seashells

April 6, 2009 • http://mathforum.org/funpow/

Welcome!

This packet contains a copy of the problem, the "answer check," our solutions, teaching suggestions, and a problem-specific scoring rubric. The problem is new, so we have no student work to share.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to "PoW Members" or use this URL to go to *funpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=526 [Log in using your PoW username/password.]

The Problem

She Counts Seashells gives students a chance to practice what they know about multiples in a context that gives meaning to them. Solving the **Extra** depends a correct solution to the main problem and extends students' thinking about multiples.

The text of the problem is included below. A print-friendly version is available from the "Print this Problem" link on the current FunPoW problem page.

She Counts Seashells

Marcia collects seashells. One day she decided to count them.

- When she counted them by 2s, there was 1 left.
- When she counted them by 3s, there were 2 left.
- When she counted them by 4s, there were 3 left.
- When she counted them by 5s, there were 4 left.

What's the minimum (smallest) number of shells she can have in her collection?

Be sure to explain how you solved the problem.

Extra: Marcia noticed a pattern in the remainders when she counted the shells. She was curious, so she counted them by 6s and then 7s to find out if that pattern would continue. Use your answer to the main problem to find out what she learned.

Answer Check

After students submit their solution, we encourage them to "check" their answer by looking at the answer that we provide. Below is what they will see. You might use the accompanying questions as prompts to help students who are struggling, or to encourage those who have found a correct solution to improve their explanation.

Marcia must have at least 59 shells in her collection.

If your answer does not match ours,

- did you try acting out the problem with cubes or chips?
- did you try using a 100 grid?
- did you think about odd and even?
- did you use what you know about multiples of 5?
- did you check your arithmetic?

If you used guess-and-test, did you tell . . .

- how you decided what numbers to try?
- how you tested them?
- how you knew whether they worked or not?
- about any patterns that helped you?

If your answer does match ours,

- is your explanation clear and complete?
- did you try the Extra question?
- did you have any "Aha!" moments? Describe them.

Our Solutions

Here are examples of some ways I imagine children might solve the problem. They are not meant to be prescriptive or comprehensive. In fact, we often receive solutions from students who have used approaches we've not anticipated. I hope you will share such approaches on the *funpow-teachers* discussion board, along with any teaching strategies you found to be successful.

Strategy 1 — Using manipulatives, testing odd numbers:

I could tell from the first clue that the number of shells had to be odd. The last clue told me there had to be at least nine shells. I used Unifix cubes to make equal groups, testing for the remainders according to the clues. If I found a remainder of 2 for groups of 3, I tested for groups of 4. If groups of 4 left a remainder of 3, I tried groups of 5. I didn't test any numbers ending in 0 or 5, because they are multiples of 5 and wouldn't have a remainder. Here are some of the numbers I tried and the remainders when I grouped them in 3s, 4s, and 5s:

Groups of:	9	11	13	17	19	21	23	…	47	49	51	53	57	**59**
3	0	2	1	2	1	0	2	…	2	1	0	2	0	**2**
4		3		1			3	…	3			1		**3**
5		1					3	…	2					**4**

I stopped when I got to 59, because that gave all the right remainders. The smallest number of shells Marcia could have is 59.

Strategy 2 — Narrow search with 2 clues, and then test for 3s and 4s:

According to the first clue the number of shells has to be odd, since it's one more than a multiple of two. Numbers that are a multiple of 5 end in 0 or 5. Numbers 4 more than those would have a 4 or 9 in the ones place. We can eliminate all even numbers, so the number must end in 9. I made a list of those and tested for the remainders when grouped in 3s and 4s. When I found a number that was 2 more than a multiple of 3, I tried dividing it by 4. Here are the remainders:

Groups of:	9	19	29	39	49	**59**
3	0	1	2	0	1	**2**
4			1			**3**

Marcia has at least 59 shells.

Extra: Marcia noticed that the remainder increased each time and was always one less than the number in the group. I divided to see whether the pattern continued for 6s and 7s:
59/6 = 9 r5 The pattern works.
59/7 = 8 r3 The pattern breaks.

Strategy 3 — What if there were one more shell . . .

I noticed that each remainder was one less than the divisor for its group. That means that, if Marcia had just one more shell in her collection, the number of shells would be a multiple of 2, 3, 4, and 5, without any remainders. I knew that number would be the least common multiple of 2, 3, 4, and 5, which is 60. Therefore, Marcia must have at least 59 shells in her collection.

She could also have any other number that was one less than a multiple of 60, such as 119, 179, 239, etc.

Extra: Since 6 is a factor of 60, counting 59 shells by 6s results in a remainder of 5, continuing the pattern. Seven is not a multiple of 60. 59/7 = 8 r3. The pattern breaks.

Teaching Suggestions

She Counts Seashells can provide an opportunity to introduce or reinforce concepts of multiples and divisibility in a meaningful context. Young children can group manipulatives or skip count on a 100 grid. Many children have difficulty remembering correct uses of factor, multiple and product. Take advantage of this problem to give them practice using that vocabulary, especially in speaking.

The Online Resources page for this problem contains links to related problems in our Library and to other web-based resources: http://mathforum.org/funpow/puzzles/supportpage.ehtml?puzzle=416

Scoring Rubric

On the last page is the **problem-specific rubric** we use to assess student solutions. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on any problem page. Please share it with your students to help them understand our criteria for good problem solving and communication.

Please let me know if you have ideas for making these packets more useful.

~ Claire

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The Math Fundamentals Problem of the Week Scoring Rubric — **She Counts Seashells (posted 6 April 2009)**
For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands the concept of equal groups with remainders. Understands that the number of shells in the collection remains constant. Makes a reasonable attempt to find the minimum number of shells Marcia can have.	Solves the main problem correctly. Understands and solves the Extra. Achieves at least Practitioner in Strategy.
Strategy *(NB: based on their interpretation of the problem)*	Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that makes sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically. OR Strategy is not evident. Might verify a correct answer, but fail to explain how they found it.	Approaches problem systematically. Applies a sound strategy, achieving success through skill, not luck. Any guess-and-test approach must reflect reasoned decisions. Chosen strategy accounts for any answer(s) that changed after checking our answers.	Does **one or more** of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology (spreadsheet) or algebra.
Accuracy *(NB: based on chosen strategy)*	Has made many errors. OR Shows/describes no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic or record keeping mistakes.	Not available for this problem.
Communication				
Completeness *(NB: an incorrect solution can be complete)*	Writes very little to explain how the answer was achieved.	Provides explanation but does not include calculations; OR Shows calculations without explanation about why they were done.	Explains most of the steps taken to solve the problem and the rationale for them, with enough detail for another student to understand. Includes numbers and key calculations. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Does **one or more** of these: Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight. Demonstrates that the answer works. Includes a relevant table of data.
Clarity *(NB: incomplete and incorrect solutions can be explained clearly*	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Uses level-appropriate math language and correct units: shells. Shows effort to use good formatting, organization, spelling, grammar, typing. Errors don't interfere with readability.	Formatting makes ideas exceptionally clear. Answer is very readable and appealing, might include a helpful diagram or image. (A diagram or image alone doesn't qualify for Expert status.)
Reflection (See list below.)	Does nothing reflective.	Includes one reflective thing.	Includes two reflective things.	Includes 3 or more reflective things or does an exceptional job with 2 of them.
	The items to the right are considered reflective, and could be in the solution OR in the comment a student leaves after viewing our answer:	• **Revises and improves the submission**. • Checks the answer using a different method. • Explains a hint she/he would give someone.	• Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them. • States any assumptions made in the solving process. • Describes something learned from the problem.	• Comments on AND explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used. • Describes any "Aha" moments.