# Problem Solving and Communication Activity Series

## http://mathforum.org/pow/support/activityseries/

This new Math Forum program is designed to help students expand their repertoire of ways to approach challenging problems. Over the course of the year, we're covering many of the fundamental problem solving strategies in conjunction with the Problems of the Week. Each problem focuses on a particular strategy, and we've provided a document that describes activities to do with your students and examples of typical student responses to the problem. The problem-specific documents both provide illustrations of the activities and help you anticipate ideas that might come up in your class.

**Round 0: Introducing the Activity Series**
The introductory document contains a series of activities designed to get students thinking about what good problem solvers do, and how to communicate their thinking by writing (or talking) mathematically.

**Rounds 1 and 2: Understanding the Problem**
What does it mean to fully understand a problem, and how does it help students find solution paths and build confidence? Included in these documents are several activities that support students to develop strategies for understanding challenging math problems, along with facilitation suggestions for teachers.

**Round 3: Guess and Check**
Guess and check is an important (and popular) problem-solving strategy, though it often gets a bad rap and may not be developed into the strong and powerful resource it could be. The guess and check strategy has at least three purposes: (1) to understand a problem thoroughly, (2) to home in on a solution, and (3) to discover efficient ways to jump to a solution by noticing patterns and developing related algebraic representations.

**Round 4: Solve a Simpler Problem**
Solve a Simpler Problem is a technique that can be used in several ways to solve challenging problems. In some situations you can see how to work the problem with easier numbers. This may show you an approach that you can try with the more difficult numbers. Second, you can choose to break the original problem into smaller steps, finding answers for parts of the problem, and then putting those together for the whole solution. Finally, students may see a way to change this hard problem into one that they have solved before.

**Round 5: Making a Table**
The Tables and Patterns strategy is a way to organize your problem solving that makes it easier to explore patterns in the calculations and results. It is often used after some initial work on the problem using Understanding the Problem or Guess and Check strategies. Tables can be used to efficiently home in on answers, or you can use tables to organize the logic of your calculations and make explicit the relationships between quantities in the problem. As you may have seen in the Simpler Problem strategy, tables can help put different iterations in order and compare them. Tables can take the form of simple t-tables to very complex spreadsheets. Spreadsheets and other related software are especially efficient because they can be used to rearrange your work for different comparisons without having to write it all over again.

**Rounds 6 through 9 repeat Rounds 2 through 5**

**Round 10: Cases**
Case-based reasoning helps problem solvers to understand the problem, work towards a solution, surface interesting mathematics, and verify the robustness of their solutions. To understand the problem, problem solvers might test interesting or representative cases and think about the different outcomes they see. When solving the problem, they might use cases to consider when certain outcomes will occur, or to narrow down the possibilities they have to investigate. Some problems have different answers for different cases. Exploring different cases can lead to questions that problem solvers might explore further, like, "what would happen if I used a negative number?" or, "would this work for obtuse triangles, too?" Finally, when determining whether a possible solution is correct, good problem solvers test their solution using multiple cases, especially cases that they know behave differently.

**Round 11: Logical Reasoning**
Logic is an inherent part of the mathematical problem solving process and was used in some ways through our Activity Series already. However, some problems depend more on logic than on purely mathematical manipulations. Logic can help us find solutions when it looks as if we are unable to solve them based on our equations. Even with problems that are primarily solved through calculations, the questions and techniques of logical reasoning can help us organize and find efficient solution approaches to problems. In this sense Logical Reasoning is particularly useful in combination with approaches such as "Noticing and Wonderings" or "PoW IQ" from Understanding the Problem (Rounds 1, 2, or 6).

**Round 12: Change the Representation**
All math problems, whether they are word problems, arithmetic problems, equations to solve, etc., come to us in a particular representation. Word problems are represented in story form, using words and often referencing a particular context. Arithmetic problems are represented numerically. Equations are represented using mathematical symbols. Each representation has benefits to the problem solver. For example, word problems allow students to apply their knowledge of the given context, which can allow them to check that their approaches are reasonable. Numeric and symbolic representations can make it easy for students to manipulate objects in the problem, and to quickly see patterns. Visual and physical representations, such as manipulatives, diagrams, and graphs, can often help students gain new insights into the problem and provide them with additional tools for solving it. Changing the representation can mean use of a different form of representation (e.g. using a line drawing for a word problem) or it can mean trying different ways of presenting the information in the same form (e.g. rewriting all of the numbers as fractions with a numerator of 1). Considering multiple representations and choosing representations that fit the problem well are important problem-solving skills.

**Round 13: Make a Mathematical Model**
A mathematical model is a way to describe a situation, usually real-world, using numeric and mathematical relationships. Mathematical models usually have inputs, operations on those inputs, certain parameters or constants that make the operations fit the particular situation, and outputs that result from performing the operations on the inputs. Sometimes in problem solving, coming up with the mathematical model to use is at the heart of the problem. Problem-solvers are engaged in noticing quantities and relationships, selecting operations to describe the relationships, and fitting those operations to the specific scenario by setting parameters. Other times, the operations and relationships are given in the problem, and the problem solver's job is to organize the information and apply it to determine a final answer. In either case, identifying quantities and relationships, and recording information as mathematically as possible are key components of making mathematical models.

**Round 14: Working Backwards**
Working backwards is a particularly useful problem-solving strategy when you can clearly define the goal or end state of the problem, and you know a sequence of operations that were used in the problem. Reversing the operations and working backwards from the goal helps problem-solvers to describe the initial conditions or the most efficient path to the goal state. Working backwards is often applied to logic problems, like the famous one about crossing the river with a cabbage, a goat, and a wolf, in which you know the goal state (everything on the opposite side of the river), and you know what the legal moves are (rowing one animal across the river without leaving the cabbage with the goat or the wolf with the goat). You can work backwards from the goal, asking yourself, what must have been the last animal rowed across the river? What must have happened just before that?

**Round 15: Planning and Reflecting**
As students get comfortable with more strategies, they begin to recognize and use multiple approaches in their problem-solving process. Following Polya (1945), it is common to break the problem-solving process down into four phases: Understanding the Problem, Making a Plan, Carrying Out the Plan, and Checking/Reflecting. Two of the most common issues in problem solving are (1) forgetting part of the problem or the ideas you have discovered that might be useful and (2) getting stuck, trying the same thing over and over without making new progress. Planning and reflecting can help you both solve problems and learn from your experience. What does it mean to get good at making a plan? How do I know when to start carrying out the plan? How do I know if I'm on a dead-end path? How do I effectively check my work?

**Round 16: Getting Unstuck**
If you never get stuck, then you are not solving interesting enough problems. Getting stuck (and, we hope, getting "unstuck"), is at the heart of problem solving. Challenging problems require that they be represented in many ways, approached with a variety of strategies, and checked again and again and again.

**Round 17: Play**
When students do the Noticing/Wondering activity, we often have them try to group their noticings into "quantities" and "relationships". With a little practice, students get adept at finding the quantities and the relationships that are explicitly stated in the problem. However, interesting math problems usually have deeper layers of relationships that only emerge as problem solvers "play" with the relationships and quantities.

Round 17 focuses on some of the ways problem solvers play with relationships and explore patterns before they delve deeply into a single problem-solving strategy. In order to make clear different aspects of problem solving, we've broken the "play" process out somewhat artificially - expert problem solvers move back and forth fluidly between understanding the problem, playing with relationships, and carrying out strategies. However, for purposes of illustration, we think it will be useful to focus on those phases separately.

**Rounds 18 revisits Change the Representation**

**Rounds 19 and 20 are to be announced!**