Pre-Algebra PoW Packet

Postal Rates

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Welcome!

This packet contains a new problem, the "answer check," our solution and scoring rubric, a note about possible common misunderstandings and mistakes we may see, and ideas for implementing the problem in the classroom.

Each of the elements in this packet are described below in more detail. Check out the PoW discussion groups to examine these topics with colleagues from around the world. When you access the Teacher Office, use the link to "PoW Members" or use this URL to go to *prealgpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=527 [Login to the discussions using your PoW username/password.]

The Problem

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Postal Rates started as an idea by Math Forum employee Richard Tchen. He drafted a problem about stamps when he realized that he had bought a postage stamp by paying with exactly one each of four different coin denominations (penny, nickel, dime, quarter). He also learned that the United States Postal Service plans to raise the price of first class mail weighing one ounce or less in May. We decided to focus on percentage increase for this PreAlgPoW.

Percentage increase might be a new concept to many of your students. Here's a thread from our Ask Dr. Math archives that might be useful to you in providing them with some examples and direction:

> Date: 06/07/2003 at 23:03:06
> From: Chief
> Subject: Percentage increase
>
> Find the percentage increase: old, $50.50; new $75.75.

> Date: 06/08/2003 at 01:28:55
> From: Doctor Mike
> Subject: Re: Percentage increase
>
> Hello,
>
> You first figure out the amount of increase. That is 75.75 minus 50.50, which is 25.25.
>
> When you deal with percentages, you compare the increase with the original, which in your case is 50.50. (original = old)
>
> For me, it has always been easier to understand this comparison step by putting it in terms of a fraction before doing the percentage version of the answer. The fraction is
>
> $$\frac{\text{increase}}{\text{before}}$$
>
> For this problem, the numbers for it are
>
> $$\frac{25.25}{50.50}$$
>
> This turns out to be exactly 0.5 or 1/2.
>
> Finally, the percentage is the fraction multiplied by 100. 100 times 0.5 is 50, so the answer is 50 percent.

> Put into words, it is a 50 percent increase to go from 50.50 to 75.75. Here is a similar problem where the numbers involved are easier. You should do the work to convince yourself that it is also a 50% increase to go from 50 to 75. You are adding 25 (half of 50 = 50 percent of 50) to get from 50 to 75. See? Good!
>
> Here is another useful related example. If you go from 50 to 100 you double the amount. That is, you add 100% of the original to get the new. This is a 100% increase.
>
> - Doctor Mike, The Math Forum

Using the simpler examples that Dr. Math suggests might be a good way to introduce your students to the topic.

As you view the data at the National Postal Museum, we are sure you will agree that there are many other interesting ideas to explore! For example, how many other times could you have purchased stamps with exactly one each of the different coins? Another idea might be to explore which increases resulted in the same percentage increase, and to connect that to the idea of equivalent fractions and the fact that they represent the same percentages.

For each problem, we will pick one category from the scoring rubric (see below) on which we'll focus. For *Postal Rates* we're choosing "Interpretation," which basically means to interpret the problem correctly and attempt to solve all of the parts. A Practitioner in Interpretation for this problem would seem to understand how to calculate percentage increase, understand that the question is only about those five specific increases, and attempt to answer the question.

The text of the problem is included here, though if you'd like a nice version to print, use the "Print this Problem" link when viewing the full problem on the website.

Answer Check
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After students submit their solution, they can choose to "check" their answer by looking at the answer that we provide. Along with the answer itself (which doesn't say anything about *how* to actually get the answer), we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Our Solution
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Previously, "our solutions" were only available to the mentors. We believe that actual student solutions are the best examples of how kids solve problems. Sometimes, however, having the actual solution ahead of time can be helpful, so we are now including it in this packet. Included in the solutions are how to support students in different areas of their work or how to think about different parts of the problem. In many instances, we also include multiple ways to solve the problem.

Scoring Rubric
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The problem-specific rubric is something we write for every problem for use by those who are assessing student work. It spells out what we expect from students in three areas of problem solving and three of communication. The goal is to look at each category separately when evaluating the student work. This way the assessment process provides more focused information regarding the areas of strength and weakness in the student work. It's important to keep in mind, however, that some of the categories affect others. For example, incomplete or unclear communication can lead to lower scores in strategy because it's harder to understand what the strategy was if it's not explained well.

Keep in mind also that a "Novice" score is not indicative of no work or a zero. It simply indicates that the student is at a beginning level in that category.

Possible Common Mistakes
right here!

Without previous student solutions, it's difficult to predict what students will have trouble with but since this problem depends on knowing "how" to find the percentage increase, that will probably be the main sticking point. As mentioned earlier, students may either need an introduction to or a reminder for this concept. In addition to the Dr. Math thread quoted earlier, another thread is referenced below. You might consider projecting them for the class or printing them on paper for groups of students to view. The earlier thread is more informal while this next one uses the idea of a "formula."

Formula to Calculate Percent Increase
http://mathforum.org/library/drmath/view/67764.html

Good luck!

We're excited about providing these new resources to you. We hope to get feedback and ideas from you on the prealgpow-teachers discussion group starting April 7, 2008.

Suzanne and *Annie*

Problem

Postal Rates

Over the years, the cost to mail the most common type of letter in the United States has gone up. The price is scheduled to increase again by 1 cent on May 12, 2008.

The National Postal Museum lists these past postal rates:

Effective Date	Cost (cents)
August 1, 1958	4
January 7, 1963	5
January 7, 1968	6
May 16, 1971	8
March 2, 1974	10
December 31, 1975	13
May 29, 1978	15
March 22, 1981	18
November 1, 1981	20
February 17, 1985	22
April 3, 1988	25
February 3, 1991	29
January 1, 1995	32
January 10, 1999	33
January 7, 2001	34
June 30, 2002	37
January 8, 2006	39
May 14, 2007	41

Question: On which of these dates was there the largest percentage increase in stamp price over the previous price?

January 7, 1963
January 7, 1968
January 10, 1999
January 7, 2001
May 12, 2008

Extra: Consider all of the increases since 1958 instead of just the five listed above. Which was the largest percentage increase?

Answer Check

The largest percentage increase was on January 7, 1963.

If your answer does **not** match our answer,

- did you need help in remembering how to find the percentage increase? Look at these pages in Ask Dr. Math: Finding Percentage of Increase or Formula to Calculate Percent Increase
- have you checked your arithmetic?

If any of those ideas help you, revise your answer, and then leave a comment that says why you revised.

If your answer **does** match ours,

- did you try the Extra?
- are you confident that you could solve another problem like this successfully?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find and fix them?
- what hints would give another student trying to solve this problem?

Our Solution

The key concepts in this problem is percentage increase.

Method 1: Reasoning It Out

First I took the important information out of the chart, since we don't need the whole chart. We only have to look at five dates. I made a list of them and the increases on those dates. The last date wasn't in the chart, but was given in the text of the problem.

January 7, 1963, 4 → 5 cents
January 6, 1968, 5 → 6 cents
January 10, 1999, 32 → 33 cents
January 7, 2001, 33 → 34 cents
May 12, 2008, 41 → 42 cents

Hey! All the increases are by a penny! That's interesting.

Percentage increase is calculated by dividing the increase by the original price. For example, if something cost $1.00 and the price went up to $1.25, the percentage increase would be 25%, since 25 is 25% of 100. I calculated the percentage increase for each of the 5 increases.

1 cent out of 4 is 1/4, or 25%
1 cent out of 5 is 1/5, or 20%
1 cent out of 32 is 1/32, or 3.1%
1 cent out of 33 is 1/33, or 3.0%
1 cent out of 41 is 1/41, or 2.4%

The first one is the biggest. That makes sense, since the denominator of the fraction is getting bigger each time, and 1 is a smaller percentage of larger numbers than of smaller numbers.

Extra: We could calculate all percentage increases, but that's tedious. (You might, of course, type the info into a spreadsheet and let the computer do the work, which is a little less tedious.) Instead, in looking at the answer to the first part of the problem, I noticed that the largest percentage increase for a price increase of 1 cent occured the first time it goes up 1 cent. So I don't even have to calculate the later percentages increases for 1 cent. I know that as the denominator gets bigger (in this case, the price before the increase), the percentage increase gets smaller.

First I wrote down the differences for all of the price increases (actually, I had the computer do it). See the table on the right. Then I calculated the percentage increase for each of the first changes of a particular amount.

Effective Date	Cost (cents)	Increase	%
August 1, 1958	4	0	
January 7, 1963	5	1	25
January 7, 1968	6	1	
May 16, 1971	8	2	33
March 2, 1974	10	2	
December 31, 1975	13	3	30
May 29, 1978	15	2	
March 22, 1981	18	3	
November 1, 1981	20	2	
February 17, 1985	22	2	
April 3, 1988	25	3	
February 3, 1991	29	4	16
January 1, 1995	32	3	
January 10, 1999	33	1	
January 7, 2001	34	1	
June 30, 2002	37	3	
January 8, 2006	39	2	
May 14, 2007	41	2	

The first change of 1 is from 4 cents to 5. That's 1/4, or 25%.

The first change of 2 is from 6 cents to 8. That's 2/6, or 33.3%.

The first change of 3 is from 10 to 13. That's 3/10, or 30%.

The first change of 4 is from 25 to 29. That's 4/25, or 16%.

So the largest percentage increase ever was from 6 cents to 8, which was a 33% increase.

Method 2: Using The Formula

Some students might find a "formula" for percentage increase that looks something like this:

$$\frac{\text{new} - \text{original}}{\text{original}} * 100 = \text{percentage increase}$$

We're not going to include an example of such a solution here. It's fine to use such a formula, but it's worth looking closely at why it works and how it's related to the work done in the solution shown above. It might be a good opportunity to remind students that we don't really "need" formulas to solve most math problems, but that knowing them offers us shortcuts to reasoning things out every time. Formulas can sometimes obscure the logic behind a particular concept. This is a good example of that. Using the formula is simply a mechanical process. Reasoning out the answer helps us see why the answer actually makes sense, since we are talking about quantities like, "1 cent out of 4 cents."

Pre-Algebra Scoring Rubric for *Postal Rates*

For each category, choose the level that *best describes* the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	doesn't seem to understand the question	thinks that the question is about all of the dates isn't sure how to calculate percentage increase	understands how to calculate percentage increase understands that the question is only about those five dates attempts to answer the question	is at least a Practitioner in Strategy and has successfully answered the Extra
Strategy	does not have any ideas about how to solve the problem	does guess and check without being careful – might have gotten lucky calculates only some of the percentage increases and makes a conclusion without explanation	has a strategy that relies on skill, not luck calculates the percentage increase for each of the five dates and compares	understands that, with a fixed numerator, the larger the denominator, the smaller the percentage increase
Accuracy	has made many errors	makes a few errors that lead to an incorrect answer	makes no arithmetic mistakes that really matter	[not normally available for this category]
Communication				
Completeness	has written nothing that tells you how they found their answer	shows work without an explanation *or* explains everything without showing the numbers doesn't include enough information for another student to follow	attempts to explain all of the steps taken to solve the problem, which might include: • where any fractions came from • how fractions are changed to percents	adds in useful extensions and further explanation of some of the ideas involved (for example, how fractions are changed to percents)
Clarity	explanation is very difficult to read and follow	another student wouldn't be able to follow their explanation entirely long and written in one paragraph lots of spelling errors/typos	explains all of the steps mentioned in such a way that another student would understand makes an effort to check their formatting, spelling, and typing (a few errors are fine)	formats things exceptionally clearly answer is very readable and appealing
Reflection	*The items in the columns to the right are considered reflective, and could be in the solution or the comment they leave after viewing our answer:*	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used	comments on and explains the ease or difficulty of the problem **revising their answer and improving anything**
	does nothing reflective	does one reflective thing	does two reflective things	does three or more reflective things or an great job with two