Pre-Algebra PoW Packet

Totolospi

March 10, 2008 • http://mathforum.org/prealgpow/

Welcome!

We have been thinking a lot about how we can work with colleagues to share implementation ideas for the Problems of the Week and get the most out of the experience, both for our students and for ourselves as professionals. One of the ideas we came up with was this packet.

It contains a problem from our Library, the "answer check," our solution and scoring rubric, a note about common mistakes we have seen, and ideas for implementing the problem in the classroom. But perhaps best of all, using a past Problem of the Week allows us to include a range of student solutions from our archives.

Reflection on student explanations has enabled us to attend more to the ways in which our students learn and to their emerging concepts rather than just to what they know. In the process we learn more math and gain a rich set of ideas for individualized instruction. Our hope is that these packets will provide a platform for all of us to share such strategies and work together to realize the power of problem-solving and communication in driving the learning process in the math classroom.

We hope that you find this useful and can apply it as you introduce each new problem to your students. Each of the elements in this packet are described below in more detail. Check out the PoW discussion groups to examine these topics with colleagues from around the world. When you access the Teacher Office, use the link to "PoW Members" or use this URL to go to *prealgpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=527 [Login to the discussions using your PoW username/password.]

The Problem
page 3

Totolospi is Problem 3716 from the Library. We've chosen it because it deals with probability. In a few weeks we'll be at the National Council of Teachers of Mathematics (NCTM) Annual Meeting in Salt Lake City. Their theme this year is "Becoming Certain About Uncertainty" and *Totolospi* will fit nicely with that theme.

For each problem, we will pick one category from the scoring rubric (see below) on which we'll focus. For *Totolospi* we're choosing "Interpretation," which basically means to interpret the problem correctly and attempt to solve all of the parts. A full range of interpretation, from Novice to Expert, can be seen in the sample student solutions included in this document.

The text of the problem is included here, though if you'd like a nice version to print, use the "Print this Problem" link when viewing the full problem on the website.

Answer Check
page 3

After students submit their solution, they can choose to "check" their answer by looking at the answer that we provide. Along with the answer itself (which doesn't say anything about *how* to actually get the answer), we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Our Solution
page 3

We've not made "our solution" available to anyone but mentors in the past, feeling that the student solutions are much better examples of how kids actually solve the problem! But sometimes having the solution ahead of time can be helpful, and we often include tips for mentors about how to support students in different areas of their work or in thinking about different parts of the problem. In many instances we also include multiple ways to solve the problem.

Scoring Rubric
page 5

The problem-specific rubric is something we write for every problem for use by those who are assessing student work. It spells out what we expect from students in three areas of problem solving and three of communication. The goal is to look at each category separately when evaluating the student work. This way the assessment process provides more focused information regarding the

areas of strength and weakness in the student work. It's important to keep in mind, however, that some of the categories affect others. For example, incomplete or unclear communication can lead to lower scores in strategy because it's harder to understand what the strategy was if it's not explained well.

Keep in mind also that a "Novice" score is not indicative of no work or a zero. It simply indicates that the student is at a beginning level in that category.

Sample Solutions
page 6

The sample student work included in this packet represents a broad range of both writing and problem solving skills. However, the biggest difference you will see in the solutions is the amount of explanation. It varies from very little to a whole lot! We want students to experience the ways in which they self-correct and have more insights as they write out more of their thinking. Sharing our selected solutions with your students after they have written their own first drafts is an opportunity to discuss what they value in someone else's explanation, leading them to add more detail in their revision. Students often don't include much information because they don't know what's possible or expected. Showing them a wide variety of student work will let them see what other students are doing, and where they could go.

In addition to providing the solutions, we've included the "Interpretation" rubric score and our thoughts about the students' solutions.

Common Mistakes
right here!

A common mistake with this problem was to only consider half of the possible outcomes. A number of students found that there were only four possible outcomes for the rolls of the dice - all round, all flat, one round and two flat, or one flat and two round. While those are all of the possible results, it doesn't represent all of the possible ways in which you could get those results. Here is a simpler example that might help students realize their mistake:

> Let's flip two coins and figure out all the possible outcomes. We could get two heads, two tails, or one head and one tail. That's three. But there are actually four different ways we could get those three ways. Let's say that one coin is a penny and the other is a dime. Here are the possible outcomes:

penny	dime
heads	heads
heads	tails
tails	heads
tails	tails

> Now the four different outcomes are easier to see. The same is true with our dice, in that there are four different end results, but eight different ways we could get those results.

Good luck!

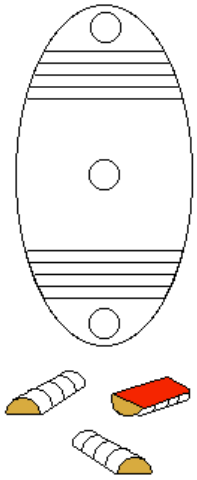
We're excited about providing these new resources to you. We hope to get feedback and ideas from you on the algpow-teachers discussion group starting March 10, 2008.

Suzanne

Problem

Totolospi

The Hopi Indians invented Totolospi, a game of chance. The game is played with three cane dice, a counting board, and a counter for each player. Each cane die can land round side up (r) or flat side up (f).



The moves of the game are determined by tossing the three cane dice with these rules:

Toss	Move
three round sides up (rrr)	player advances 2 lines
three flat sides up (fff)	player advances 1 line
any other toss of the three cane dice	player doesn't advance

Questions:

1. List all the possible tosses (combinations of cane dice landing positions).
2. If landing round side up or flat side up were equally likely, what is the probability that you will be able to advance your counter on a toss?

Extra: Do you think it is actually equally likely for a cane die to land round side up or flat side up?

Answer Check

The probability of advancing your counter on a toss is 1/4. Don't forget to also answer the first question.

If your answer **doesn't** match ours,

- did you express the probability as a percentage? 25% is also correct.
- try using the "r" and the "f" to keep track of what could happen each time you rolled the three dice.
- try making a chart to keep things organized.

If any of those ideas help you, revise your answer, and then leave a comment that says why you revised.

If your answer **does** match ours,

- have you tried answering the Extra?
- have you clearly shown and explained the work you did?
- are you confident that you could solve another problem like this successfully?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?

Our Solution

The key concepts are probability and permutations.

Question 1: How many possible tosses?

There are a couple of ways to do this.

Method 1: Organized List

You can make an organized list that shows the different possibilities. Start with all the possibilities that have "round" (**r**) in the first position.

r r r
r r f
r f r
r f f

We know that's right because it has the **r** and **f** in the second and third position each twice. Now we put the flat side (**f**) in the first position:

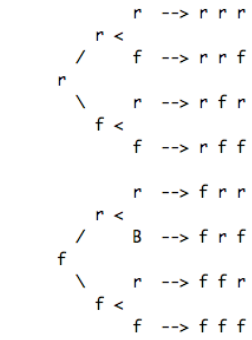
f r r
f r f
f f r
f f f

That's four more, for a total of eight possibilities.

Note: Kids might make lists of the possibilities that aren't especially organized. This might lead them to find all of the possibilities without knowing that they found them all. In order to be a Practitioner in Strategy, they have to actually have some sort of organization and to indicate that they know they've found everything.

Method 2: Make a Tree Diagram

This is a nice way to listing out all of the possibilities. The different stages of the diagram let you show all of the possibilities at each stage. Here we can see that we have a choice of **r** or **f** for each additional line.



There are eight possibilities.

Question 2: What's the probability that you'll advance?

To answer this, we need to figure out the probability that all of the dice will be the same (either "round" or "flat"). To calculate probability, we find the number of positive outcomes and divide it by the total possible outcomes.

We already figured out that there are 8 possible ways the dice could land. From that list, we can see that there are 2 ways where all three are the same (**r r r** or **f f f**). That's 2 out of 8, or 1/4 chance that the player will advance. 25% would also be acceptable.

(Doesn't sound like an exciting game if you only advance one time out of four!)

Extra: Obviously, answers to this part will vary. Our own carefully-conducted research leads us to believe that it's fair, but we don't actually *know* that it is. But if they give decents reasons and have thought about it some, that's probably sufficient.

Pre-Algebra Scoring Rubric for *Totolospi*

For each category, choose the level that *best describes* the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	doesn't seem to understand the game or doesn't answer either question	answers only one of the two questions answers both questions, but misunderstands how the game works	understands how the dice could fall and how the game works attempts to find all the combinations of tosses atttempts to find the probability of moving	is at least a Practitioner in Strategy and has provided a reasonable and somewhat thoughtful answer to the Extra
Strategy	does not have any ideas about how to solve the problem	does guess and check without being careful – might have gotten lucky not organized when finding list	has a strategy that relies on skill, not luck is organized in finding all the combinations	uses a tree diagram
Accuracy	has made many errors	makes a few errors that lead to an incorrect answer	makes no arithmetic mistakes that really matter	[not normally available for this category]
Communication				
Completeness	has written nothing that tells you how they found their answer	shows work without an explanation *or* explains everything without showing the numbers doesn't include enough information for another student to follow	attempts to explain all of the steps taken to solve the problem, which might include: • how the list of tosses was generated • how it's known the list is complete • how the probability was calculated	adds in useful extensions and further explanation of some of the ideas involved (for example, why the Counting Principle works)
Clarity	explanation is very difficult to read and follow	another student wouldn't be able to follow their explanation entirely long and written in one paragraph lots of spelling errors/typos	explains all of the steps mentioned in such a way that another student would understand makes an effort to check their formatting, spelling, and typing (a few errors are fine)	formats things exceptionally clearly answer is very readable and appealing
Reflection	*The items in the columns to the right are considered reflective, and could be in the solution or the comment they leave after viewing our answer:*	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used	comments on and explains the ease or difficulty of the problem **revising their answer and improving anything**
	does nothing reflective	does one reflective thing	does two reflective things	does three or more reflective things or an great job with two

Thomas
age 14

Interpretation
Novice

There is 24 possible tosses

Thomas mentions the word "toss" indicating that he has some idea about the context of the problem.

I wonder how he counted 24 possibilities? I might ask him to tell me 3 of those possibilities to see if he actually understands how the game is played.

Kariston
age 14

Interpretation
Apprentice

There is 9 possible tosses

I put different combinations of round up sides and flat sides up.

Kariston has responded to one of the two questions asked. I wonder if I reminded him to use "r" and "f" if he could list the 9 possible tosses that he found.

Once he has a list he might be able to think more about the second question.

Amos
age 9

Interpretation
Practitioner

The answer to the first question to the problem of the week is that the possibilities are:

RRR
RRF
RFF
FFF

I got this answer by starting at three R's. Now the next roll that could have happened that would be the closest to three R's is two R's and one F. The next closest to three R's is one R and two F's. The next is three F's. Now since I don't have any more R's in the last roll, that means that there aren't any more possible rolls.

The answer to the next question is that the chance that you have to roll three R's or three F's is a 2 out of 4 chance. I got this answer by looking at the graph that I had made to answer the problem before, and since two of those four possible rolls were three of a kind, then the chance you have to roll three R's or three F's is a 2 out of 4 chance.

The answer to the EXTRA problem is that it is not equal in chance that you will roll a R than you will a F. I think that it is harder to roll a R than it is to roll a F. I do because if it falls on an edge which there is a 50 50 chance that it does, because the F side is larger, it will weigh more, and if it weighs more, it will tip to that side. So if you had four cane dice and each one landed on one of these four possible sides: side, side, R, F. You will then notice that if all the sides land o the F side, 3 out of 4 cane dice will be F's.

Amos has a good start on his solution. It seems that he was focusing on the pattern of the decreasing Rs rather than thinking of all of the possible tosses.

I wonder if he might pick up on continuing that pattern if I asked him if FFR might come next after FFF. I wonder if he would think that was different from RFF?

Tyler
age 12

Interpretation
Practitioner

1) There are four possible tosses (rrr,fff,rfr,and ffr).
2) The probality of you advancing the counter is 1:2

1) To get the answer of four possible tosses I first drew the two that were discibed in the graph (rrr and fff). Then I knew there were more possible tosses because on the graph the third row said any other toss of the three cane dice the player doesnt advance. So I drew two other possibilities (rrf and ffr). Then when I tried to draw other tosses they were just rrf and ffr rearanged like rfr or frf.

2) The probability would be 1:2 or 50% chance of moving forward if landing round side up or flat side up were equally likely because there are four possible tosses and only two of the four let you advance. Then I knew that the probability of advancing is 1:2 or 50%.

Tyler like Amos has listed only 4 of the 8 possible tosses. Unlike Amos, however, Tyler states that he has rearranged rrf from rfr.

I wonder if it would help Tyler to share the penny/dime example that we've mentioned in this packet?

Claire
age 14

Interpretation
Practitioner

8 possible tosses, 1:4 probability to advance

rrr, fff, rfr, frf, rrf, ffr, rff, frr are the possible tosses.

2 choices out of the 8 will let you advance and 2:8 = 1:4 probablilty

Claire seems to have interpreted the problem well. She has stated her answers but she hasn't really explained how she knows.

Archit
age 11

Interpretation
Practitioner

1. The possibilities are;
   fff
   ffr
   frr
   frf
   rrr
   rrf
   rff
   rfr
There is a total of 8.
2.There is a 1/4 chance of advancing a space.

1. To solve, I made an Organized list with f being lanading on the flat side and r landing on thge round side;
   fff
   ffr
   frr
   frf
   rrr
   rrf
   rff
   rfr
There are a total of 8 combinations.

2. According to the rules of Totolospi, one advances only when it is rrr or fff.
In the list above, rrr and fff account for 2/8 of the possible outcomes.
2/8 simplifies to 1/4, or .25, or 25%.

EXTRA: I do not think it is equally likely to roll a flat or round.
I believe that the mass of the cane die would pull it down a certain way more often.
Also, because the round surface has more area than the falt surfae, a random;y thrown die will
more likely land on the round side making it have the flat side face up.

Archit, unlike Claire, explains the notation he used to make his organized list. He also refers back to the rules of Totolospi and refers specifically to which tosses are possible outcomes.

While both Archit and Claire interpreted the problem at the practitioner level, Archit would receive higher scores in communication.

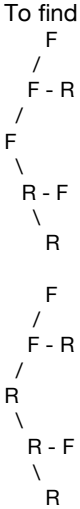
Channing
age 12

Interpretation
Practitioner

The possible tosses are fff, ffr, frf, frr, rff, rfr, rrf, and rrr.
The probability that you will advance your counter on a toss in 1/4.

Extra: I do not think it is equally likely for a cane die to land round side up or flat side up.

To find the possible tosses, use a tree diagram:



Going down, the combinations are f-f-f, f-f-r, f-r-f, f-r-r, r-f-f, r-f-r, r-r-f, r-r-r, a total of 8 combinations.

If landing round side up or flat side up were equally likely, you would compute the probability of advancing as follows: since two combinations allow you to advance (fff, rrr), out of eight possible combinations, as shown above, the probability is 2/8, or, simplified, 1/4.

Extra: It is not equally likely for a cane die to land round side up or flat side up becuase the surface area of the round side is greater, and the difference in the shapes of the sides cause the aerodynamics of the sides to be different.

Channing's tree diagram and the explanation that accompanies it demonstrate a clear understanding of this problem.

Thomas
age 13

Interpretation
Expert

The possible tosses are rrr, rrf, rff, fff, ffr, frr, frf, rfr. The probability is 25%.

To find all of the outcomes, I first found how many possible outcomes there were. To do this I did 2^3 because there are two possible outcomes for each toss and 3 tosses.

2^3=8

Since I knew there were 8 outcomes, I just wrote down possible outcomes until I found 8 different ones.

To find the probability of advancing, I looked at the possible outcomes and since there were 8 total outcomes and only 2 outcomes make you advance, I made a fraction.

2/8=1/4

Since I got 1/4, I changed it to a percent.

1/4=25%

I found that the probability of advancing is 25%.

Extra: No, I do not think it is equally likely for a cane die to land round side up or flat side up. This is because if it at first lands flat side up, there is a chance of it rolling over to make it round side up. Therefore, there is a greater probability of the cane die landing round side up than flat side up.

Thomas knows how to calculate possible outcomes but he carefully confirmed that he could list them all.

I wonder if he could explain how he knew to do that?