Math Fundamentals Problem Packet for Teachers:

Siblings

March 31, 2008 • http://mathforum.org/funpow/

Welcome!

This packet contains a copy of the problem, the "answer check," our solutions, teaching suggestions, and a problem-specific scoring rubric. ***Siblings*** is a new problem, so there will be no sample student solutions, as there are with problems we republish from the Library.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to "PoW Members" or use this URL to go to *funpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=526 [Log in using your PoW username/password.]

The Problem

The arithmetic required by ***Siblings*** is minimal. I expect most solvers to apply logical reasoning with a guess-and-test approach. Those who use guess-and-test should keep a record of their trials and to report the details of their process (see suggestions in the Answer Check below). They should also be encouraged to notice things along the way that can lead to a solution more efficiently. Older students might use algebraic or pre-algebraic techniques.

Solving the Extra successfully depends on a correct solution to the main problem. It's an opportunity for students to apply their conceptual understanding of fractions.

The text of the problem is included below. A print-friendly version is available from the "Print this Problem" link on the current FunPoW problem page.

Siblings

Alex has the same number of brothers as he has sisters. His sister Megan has twice as many brothers as she has sisters. How many children in the family are boys and how many are girls?

Explain how you found the answer. Show how you can tell that you are right.

Extra: At a family reunion Alex and his siblings represented 1/4 of all the children there. How many children attended the reunion?

Explain how you found the answer.

Answer Check

There are four boys in the family. Now you can figure out how many girls there are.

If your answer does **not** match ours,

- think about what the first sentence tells you about the number of girls and boys in the family.
- did you try acting out the problem with people or objects?
- did you make a table to keep track of what you tried?
- did you check your arithmetic?

If you used guess-and-test, did you tell . . .

- what numbers you tried?
- how you tested them?
- how you knew whether they worked or not?
- how you decided what to try next?

If your answer **does** match ours,

- is your explanation clear and complete?
- did you try the Extra question?
- did you verify your answer with another method?
- and you made a correct first guess, show how you would test a different guess.

Our Solutions

Here are three examples of some ways I imagine children might solve the problem. They are not meant to be prescriptive or comprehensive. In fact, we often receive solutions from students who have used approaches we've not anticipated. Those are cause for celebration! I hope you will share such approaches on the *funpow-teachers* discussion board, along with any teaching strategies you found to be successful.

Strategy 1 – Using manipulatives (or classmates) and logical reasoning:

I used blue cubes for boys and red cubes for girls. I put a sticker on a blue one to represent Alex and another on a red one for Megan. These helped me think about the problem and see the relationships in the family.

Since Alex has the same number of brothers as he has sisters, I knew that the number of boys has to be one more than the number of girls. Alex is a boy, but doesn't count as one of his own brothers. He has at least one sister, Megan.

I also knew that the number of boys must be even, because Megan has twice as many brothers as sisters, and twice any number must be even. The number of girls, one less than the number of boys, must be odd, and more than one, since Megan must have at least one sister.

The smallest number of girls possible would be 3. That would mean that there would have to be 4 boys. Alex would have 3 sisters and 3 brothers. Megan would have 4 brothers and 2 sisters. That meets all the conditions in the problem, so I know there are 4 boys and 3 girls.

Strategy 2 – Make a table with guess-and-test:

I made a table. I tested numbers of boys and girls, starting with the lowest possible, and seeing if each pair fit the facts of the problem. Since Alex has the same number of brothers as sisters, I knew that the number of boys must be one more than the number of girls. I tried pairs of consecutive numbers.

Boys	Girls	Megan's sisters	Megan's brothers 2x her sisters?
2	1	0	no
3	2	1	no
4	3	2	yes! 4/2 = 2
5	4	3	no
6	5	4	no
7	6	5	no

There are 4 boys and 3 girls in the family. Alex has 3 brothers and 3 sisters. Megan has 4 brothers and 2 sisters.

Note: Older students might begin with a similar approach and reasoning, but use fractions, ratios or proportions to compare Megan's brothers and sisters. This representation may lead to the discovery that, as the number of boys and girls increases, the ratio of boys to girls decreases, helping solvers to be more confident that there are no other pairs of numbers that satisfy these relationships. Such a table might look like this:

Boys	Girls	Megan's brothers/sisters
2	1	2/0 undefined
3	2	3/1 = 3
4	3	4/2 = 2 It works!
5	4	5/3 = 1 2/3
6	5	6/4 = 1 1/2
7	6	7/5 = 1 2/5

Strategy 3 - Algebra:

I used B to represent boys and G to represent girls. Since Alex has the same number of brothers as sisters, I knew that the number of boys must be one more than the number of girls.
$B = G + 1$

Megan's sisters is the number of girls minus Megan herself, or G-1. Since Megan has twice as many brothers as she has sisters, the number of boys must be twice the number of Megan's sisters.
$B = 2 (G - 1)$

Substituting the first expression for boys into the second equation:
$G + 1 = 2 (G - 1)$
$G = 2G - 2 - 1$
$3 = G$
$B = G + 1 = 4$

There are 4 boys and 3 girls in the family. Alex has 3 brothers and 3 sisters. Megan has 4 brothers and 2 sisters.

Extra: There are 7 children in Alex's family. If they are 1/4 of the children at the reunion, then the total number of children must be 4 * 7 = 28 children.

Teaching Suggestions

Resist the urge to give direct instructions on a specific approach. The best way to coach children who are struggling is to ask questions that help them discover patterns, notice the effect of odd/even numbers, and reach a deeper and more generalized understanding of the problem. Good questions help children clarify their thinking and give you useful information as well.

Ask children to verbalize what they know about the number of girls and boys in the family from the information given in each of the first two sentences of the problem, one at a time. For example, "If Alex has the same number of brothers as he has sisters, what can we say about the number of boys and girls in the family?" "Could they be even numbers? Odd numbers?" Understanding the mathematical relationships gives them a basis for testing numbers, and helps narrow down their search.

Acting out (modeling) the problem with classmates or manipulatives may help students conceptualize the relationships in the problem. It may help to simulate the problem with diagrams, or by using actual names to represent the children.

The Teacher Support Page for this problem contains links to related problems in the Problem Library and to other web-based resources: http://mathforum.org/funpow/puzzles/supportpage.ehtml?puzzle=365.

Scoring Rubric

On the last page is the **problem-specific rubric,** to help those who are assessing student solutions. It specifies what we expect from students in three areas of problem solving and three areas of communication. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on every problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of FunPoWs. Please let me know if you have ideas for making them more useful.

~ Claire

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The Math Fundamentals Problem of the Week Scoring Rubric — **Siblings** (posted 31 March 2008)

For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands that Alex has the same number of brothers as sisters. Understands that Megan has twice as many brothers as sisters Answers the main question.	Solves the main problem correctly. Understands and solves the Extra. Achieves at least Practitioner in Strategy.
Strategy *(NB: based on their interpretation of the problem)*	Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that makes sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically. OR Verifies a correct answer, but fails to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill and understanding, not luck. Guess-and-Check approach must involve good reasoning and informed guessing. Chosen strategy accounts for any answer(s) that changed after checking our answers.	Does **one or more** of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology or algebra.
Accuracy *(NB: based on chosen strategy)*	Has made many errors. OR Shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic or record keeping mistakes.	Not available for this problem.
Communication				
Completeness *(NB: an incorrect solution can be complete)*	Writes very little to explain how the answer was achieved.	Describes the steps but does not include calculations or numbers. OR Shows calculations without rationale or explanation.	Explains most of the steps taken to solve the problem and the rationale for them, with enough detail for another student to understand. Includes key calculations with rationale. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Does **one or more** of these: Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight into the problem.
Clarity *(NB: incomplete and incorrect solutions can be explained clearly*	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Uses level-appropriate math language and notation. Shows effort to use good organization, formatting, spelling, grammar, typing. Errors don't interfere with readability.	Formatting makes ideas exceptionally clear. Answer is very readable and appealing, might include a helpful table. (A table alone doesn't qualify for Expert status.)
Reflection (See list below.)	Does nothing reflective.	Includes one reflective thing.	Includes two reflective things.	Includes 3 or more reflective things or does an exceptional job with 2 of them.
	The items to the right are considered reflective, and could be in the solution OR in the comment they leave after viewing our answer:	• **Revises and improves the submission**. • Checks the answer using a different method. • Explains a hint she/he would give someone.	• Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them.	• Comments on AND explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used.