

Discussion Questions

- What's a simple rule to calculate

$R_4(n)$ for any n ? It's n , except it's 0 when multiples of 4

$S_4(n)$ for any n ? $S_4(10) = \frac{\sigma(10)}{4}$

child of R_4

$S_4(n) =$ sum of divisors of n unless they're multiples of 4.

$$1 + 2 + 5 + 10 = 18$$

- How many ways can 120 be written as the sum of 4 squares?

Divisors of 120 are 1, 2, 3, ~~4~~, 5, 6, ~~8~~, 10, ~~12~~, 15, ~~20~~,
~~24~~, 30, ~~40~~, ~~60~~, ~~120~~

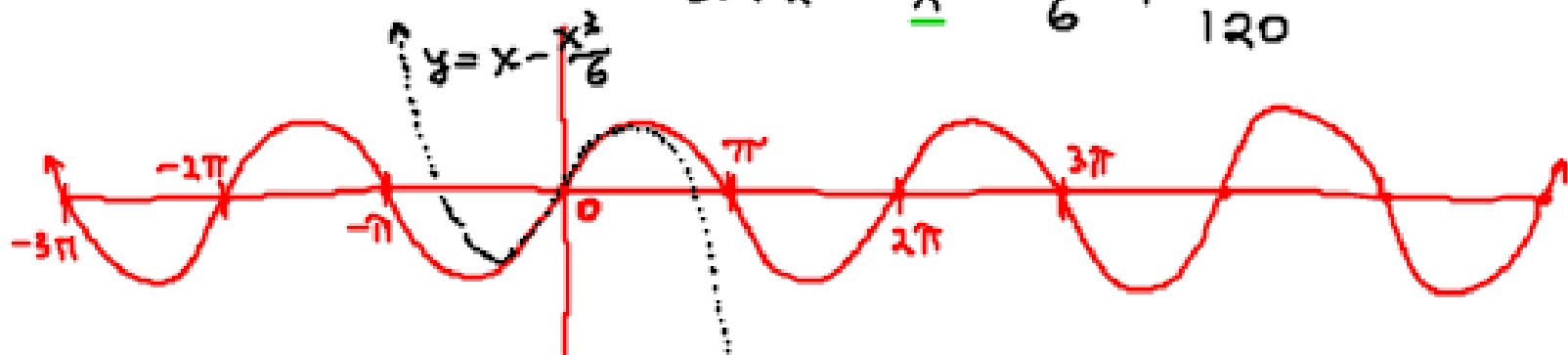
$$S_4(120) = 1 + 2 + 3 + 5 + 6 + 10 + 15 + 30 \\ = 72$$

How does this relate to $S_4(30)$?

$$s_4(120) = \# \text{ of ways} = 8 S_4(120) = 576$$

$$f(x) = \sin x$$

$$\sin x = \underline{x} - \frac{x^3}{6} + \frac{x^5}{120} - \dots \leftarrow$$



But let's use Factor Theorem instead.

$$\sin x = Ax(x-\pi)(x+\pi)(x-2\pi)(x+2\pi)(x-3\pi)(x+3\pi)\dots$$

$$= Ax(x^2 - \pi^2)(x^2 - 4\pi^2)(x^2 - 9\pi^2)\dots \leftarrow$$

$$= x\left(1 - \frac{x^2}{\pi^2}\right)\left(1 - \frac{x^2}{4\pi^2}\right)\left(1 - \frac{x^2}{9\pi^2}\right)\dots$$

$$= x - \left(\frac{x^3}{\pi^2} + \frac{x^3}{4\pi^2} + \frac{x^3}{9\pi^2} + \frac{x^3}{16\pi^2} + \dots\right)\dots$$

$$x - \frac{x^3}{\pi^2}\left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots\right) \stackrel{\therefore}{=} x - \frac{x^3}{6}\dots$$