TODAY IS FREE SLURPEE DAY!! GO GET ONE!!

5 *i, Robot*

Important Stuff

> **PROBLEM**
>
> For each of these numbers, square it, then calculate the N-value of the original number.
>
> | | |
> |---|---|
> | $2+i$ | $3+i$ |
> | $3+2i$ | $4+i$ |
> | $5+2i$ | $7+4i$ |
> | $8+3i$ | $15+4i$ |

Do you remember what N is? It's a function takes a number and multiplies it by its conjugate.

So since we're not calling this function "norm," let's call it "cliff." Cheers!

The lost numbers from Lost, a joke about robots, and a reference to Karen were lost on this problem set.

1. Describe a way to find Pythagorean triples.

2. Expand this: $(x+yi)(x-yi)$.

$(x+yi)(x-yi)$

3. Find all 12 ordered pairs (x, y) with integers x, y that have

$$(x+yi)(x-yi) = 25.$$

4. (a) Draw a graph of all the points (x, y) that are 5 units away from the origin. Mark all points on the graph that have integer coordinates.

(b) Write an equation for the graph you just (An)drew.

5. Find a and b so that $N(a+bi) = 13$. Find c and d so that $N(c+di) = 5$. Calculate $N\left((a+bi)(c+di)\right)$.

Oh yeah, a, b, c and d are supposed to be integers.

6. Use the results of problem 5 to find a Pythagorean triple with hypotenuse 65.

7. How many ordered pairs (x, y) with integers x, y are there with $N(x+yi) = 17$? What about 65? What about 169? What about ⟨*insert your favorite three-digit prime that isn't 101 here*⟩?

Feeling like a robot yet?

8. Email a title for the Day 6 problems to `dyong@hmc.edu`.

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Neat Stuff

9. Does squaring $a + bi$ give you all the Pythagorean triples?

10. Pick some other integers a and b and use your method from problem 1 to generate some triples from $a + bi$. You may find that sometimes the "leg" lengths are zero or negative. Refine your method so that this doesn't happen.

11. Determine if the Pythagorean triples that are produced by squaring these numbers in the box on the previous page have any common factors. Then, find other numbers whose corresponding triples don't have common factors.

12. Find a number n that is the hypotenuse of exactly *four* primitive Pythagorean triples.

No, a primitive Pythagorean triple is not one that is printed on papyrus. A *primitive* Pythagorean triple is one in which the three lengths don't share a common factor.

13. Find a primitive Pythagorean triple whose hypotenuse length is 13^3.

14. Prove that at least one number in every Pythagorean triple must be even.

15. In class, we conjectured that any number that is one more than a multiple of 12 can be written as the sum of two squares (of integers). Does this always work?

16. Find all possible ordered pairs (x, y) with integers x, y so that $x^2 + y^2 - xy = 91$. What about 133?

17. Write each prime as $n = x^2 + y^2 - xy$, where x and y are integers, or determine that it's impossible.

(l) 103
(a) 107
(u) 109
(r) 1009
(i) 4003
(e) 11111111111111111111111

Oh yes, there are 23 ones right there.

18. Write each number as $n = x^2 - 2y^2$, where x and y are integers, or determine that it's impossible.

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

19. Calculate this:

$$\frac{(5+i)^4}{239+i}$$

What the heck could this possibly be useful for?

20. Show us your favorite tangram. Ask Art about his favorite.

rat tat bomb

Tough Stuff

21. Let (x, y) be a point on the unit circle. If you walk along the circle from $(1, 0)$ to (x, y), then walk that same distance farther along the circle, where will you be?

This question should use trigoNOmetry. As in, don't use that.

22. Prove that every positive integer not of the form $8n + 7$ or $4n$ is a sum of three squares having no common factor.

Legend(re) proved this in 1798.

23. Suppose n is a positive integer. Is there a right triangle with rational numbers as its side lengths with area n?

24. Is there a generalization of Pick's Theorem for isometric geoboards?

This was Brian's idea. Blame him if you get stuck. Call AAA if your car gets stuck.

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