2007.5 We've Got Game

Game of the Day: "Race Game"

Alice, Bev, Craig, and Dawn sit at a table for four in no particular order. Rey, the guest of honor, tells them their seats are assigned at the table and shows them the chart.

Rey is getting an award as the King of Math.

Twelve minute time limit!

1. (a) What is the probability that *all four* of them are in the right seat?

(b) What is the probability that *all four* of them are in the *wrong* seat?

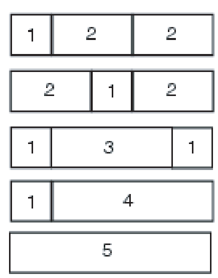
2. *On average*, how many of the four will be sitting in the right seat?

Important Stuff.

You can use rods of integer sizes to build "trains" that all share a common length.

Using Cuisenaire rods helps. Unless you are told otherwise, you have an unlimited supply of all the rod types, even beyond the 10 that is the normal limit of the Cuisenaire set.

A "train of length 5" is a row of rods whose combined length is 5. Here are some examples:



Notice that the 1-2-2 train and the 2-1-2 train contain the same rods but are listed separately. If you use identical rods in a different order, this is a distinct train.

3. (a) How many distinct trains of length 4 can be made?

(b) How many trains of length 4 can be made with one rod? two rods? three? four? five? A 1-1-2 train counts as three rods, by the way.

Five rods? Eh? Let's just say this shouldn't be that hard.

4. Repeat problem 3 for trains of length 5. The work on the previous problem might be helpful!

5. (a) Make a table for $n = 2$ to 8 for the number of trains of length n that use *exactly* two rods.
(b) Repeat for *exactly* three rods.

6. Physically make all five trains of length 6 that use exactly two rods, and (separately, without destroying the two-rod trains) all ten trains of length 6 that use exactly three rods.

7. **Calculator skill time.** Here are some explicit directions on how to evaluate $\binom{5}{2}$ on the calculators.

Not sure what $\binom{5}{2}$ means? No worries, we shall discuss.

- Hit the HOME button then select option 5 to get a new document.
- When it asks you what kind of page you want, select Calculator.
- Type out the word `ncr` then, in parentheses: `(5, 2)`. Your calculator line should look like this:

  `ncr(5,2)`
- Hit the enter key in the bottom right and enjoy the magic. The letter C will be capitalized in the display, which will appear as `nCr(5,2)`.

(a) Calculate $\binom{5}{2}$, $\binom{6}{2}$, and $\binom{7}{2}$.
(b) The calculator can work with variables, too. What does the calculator give for $\binom{x+1}{2}$?
(c) What does the calculator give for $\binom{x}{k}$?

Neat Stuff.

8. Billy is in the sixth grade, and he estimates that the books he carries to school weigh about 25 pounds. He figures he's right to within 3 pounds, give or take.

Billy's dog Woody weighs about 18 pounds, give or take 2 pounds.

It depends on how many Milk-Bones Woody's been eating lately.

(a) Billy says the books and the dog combined should weigh about 43 pounds, but how accurate could this be in "give or take" terms?
(b) One day, Billy decides to carry Woody in his school bag instead of his books. A lighter load! Give an estimate for the difference in the weights, in "give or take" terms.

9. *Make a game* that is about one-third likely to be won. Explain clearly how the game is played, and what the winning condition is. The best games are simple to play but complex in their potential outcomes. Don't worry too much about making the winning probability exactly one-third.

So, one game would be "Roll a die and if it comes up 1 or 2, you win." But you can make something more interesting!

10. Devise an experiment using one or more six-sided dice with exactly a one-*fifth* probability of success.

Ack! Stupid dice and their six sides.

11. With 240 coin flips come 120 pairs of flips.

(a) The four outcomes for a pair of flips are HH, HT, TH, TT. Given 120 pairs of flips, how many of each outcome would you expect?

(b) Find a 240-flip data set and determine the number of each paired outcome that occurred. Do you find any grounds to suspect that the data is fake?

12. What's the probability that an integer picked from 1 to n is a perfect square if

(a) $n = 10$?

(b) $n = 100$?

(c) $n = 1000$?

(d) $n = 10000$?

(e) What is happening "in the long run" (as n grows larger without bound)?

13. What's the probability that an integer picked from 1 to n is *square free* if

An integer is *square free* if it has no square factors greater than 1. So 4 isn't a factor, 9 isn't, and so on. 30 is square free; 60 isn't. You might need some technology help for this one.

(a) $n = 10$?

(b) $n = 100$?

(c) $n = 1000$?

(d) $n = 10000$?

(e) What is happening "in the long run" (as n grows larger without bound)?

14. Some of the terms in the Farey sequence have consecutive Fibonacci numbers as numerator and denominator. One such fraction is $\frac{5}{8}$, while another is $\frac{21}{34}$.

Where do these fractions appear within the Farey sequence? Can you find any justification?

15. Show that there must be a sequence of 100 consecutive non-prime positive integers.

16. On average, how far is it from one prime number to the next? How would you go about measuring such a thing?

Useless Stuff.

17. How many ways are there to use white and red Cuisenaire rods to build a cube, two units on a side? You are allowed to place red rods in any direction, including vertically.

This is a very nice problem, but we're not kidding: it's useless stuff.

Tough Stuff.

18. What's the *average* length of car used when you make all the trains of length 5? Is there a general rule at work here? Can you justify it?

19. Find a generalization to the problems in today's "Game of the Day" that can work for n people or prizes instead of 4. Specifically, what happens to the probability that everyone ends up in the wrong seat?

20. Henri and Tatyana play a very long game. They flip a coin: if it's heads, Henri gets a point. If it's tails, Tatyana gets a point. What makes it such a long game? Well, in order to win, you have to be 20 points ahead of your opponent.

How long, on average, will this game last?

21. A person is standing at the edge of a pool, and they've had one too many. Each step they take, they have a $\frac{1}{3}$ chance of stepping toward the pool, and a $\frac{2}{3}$ chance of stepping away from the pool.

Must be coming back from the Margarita-Off or something.

What is the exact probability that they eventually fall into the pool? Note that this probability will be more than $\frac{1}{3}$ since the first step could take them into the deep end.

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Well, not really, but we've already asked 21 questions and had extra space. Anybody want a spatula?