2007.3 Game On!

Game of the Day: "¿Vas o No Vas?"

A popular game show involves opening suitcases with different dollar amounts in them. Every so often, the player is offered a deal to walk away instead of continuing the game. If the player refuses all deals, they get to keep whatever is in the final suitcase.

1. There are four suitcases left: \$5, \$25, \$10,000, and \$50,000.

(a) The show offers you \$9,000 to walk away. Should you take the deal? Why or why not?

(b) What if the show offered you \$12,000? Would you take that deal? Why or why not?

2. There are five suitcases left: \$1, \$5, \$5,000, \$30,000, and \$125,000. You are a representative of the show. What deal amount would you offer, and why?

3. The actual show begins with these 26 suitcase values:

If you're expecting a \$1 million suitcase, you've got the wrong show.

\$.01	\$.25	\$1	\$5	\$10	\$25
\$50	\$75	\$100	\$200	\$300	\$400
\$500	\$1,000	\$1,250	\$2,500	\$5,000	\$7,500
\$10,000	\$25,000	\$30,000	\$40,000	\$50,000	\$100,000
\$125,000	\$250,000				

What would a *fair deal* be worth at the start of the game? Describe how the calculation was made.

Important Stuff.

4. This is G_5, the *Godmother sequence* of order 5:

This is opposed to the *Godfather sequence*, which is {I, II, III}.

$$\left\{\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1}\right\}$$

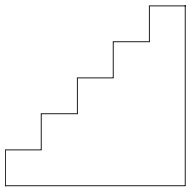
The sequence G_5 is all fractions from 0 to 1, inclusive, with denominators less than or equal to 5, and is written in increasing order with all fractions written in lowest terms.

Write out G_6, G_7, and G_8.

5. How many elements are in each of G_1 through G_8?

6. Here is a geometric shape:

This follows up on some of the geometric discussion we had Tuesday.



(a) If the length and height of each stair is 1 foot, find the area of the shape.
(b) Show how two staircases of this shape could be combined into a rectangle, and find the area of the rectangle.
(c) Suppose the staircase was 9 steps instead of 5. Could two such staircases still form a rectangle? Use this to find the area of *one* such staircase.
(d) Suppose the staircase was 75 steps. Could two such staircases still form a rectangle? Use this to find the area of *one* such staircase.
(e) Find a rule for the sum of the first n integers

$$1 + 2 + 3 + \cdots + n$$

7. Pick one of the coin sets from Tuesday that you *haven't* worked on yet, and determine whether it is real or fake. We are likely to have a discussion about the different tests people have been designing.

There were seven on Tuesday's set, three on an appendix page. We have more, just ask.

8. After the parade, Alan was stuck with a giant piece of paper. He tears it into five equal pieces and hands one piece to Caroline, one piece to Jim, one piece to Philip, one piece to RoseMary, and keeps the fifth piece for himself.

He continues to do this with his remaining piece, dividing it into five equal pieces.

(a) After two tears, how much paper does Alan have left? How much do the others each have?
(b) After three tears?
(c) After 10 tears?
(d) Forever?
(e) Write two different expressions for the amount of paper Caroline has after this is over.

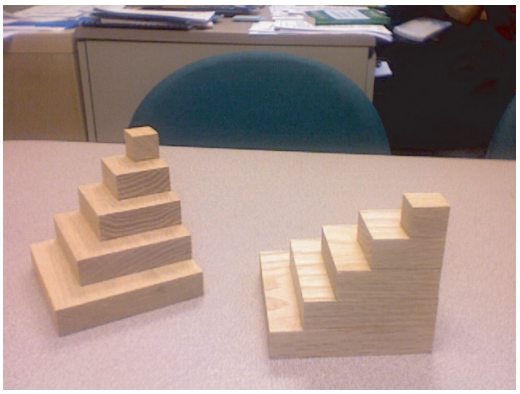
Neat Stuff.

9. On the hit show Italian Mathematical Bingo Night, the bingo machine is filled with balls for the Fibonacci numbers:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233$$

How would the player's strategy in Tuesday's "Game of the Day" be altered by this change?

10. The formula for the sum of the first n squares can be modeled with blocks of wood! Here's a picture of two blocks.



(a) Explain why the volume of each block of wood pictured above is

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

(b) Show how six blocks in this shape can be fit together to form a solid box (or, better yet, build it). Then, find the dimensions of this box.

(c) Describe how this process could be generalized to show that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

11. What's the probability that an integer picked from 1 to n is a perfect square if

(a) $n = 10$?
(b) $n = 100$?
(c) $n = 1000$?
(d) $n = 10000$?

You'll notice a few retreads from earlier sets. Skip 'em if you already did 'em, but they're still neat if you haven't seen them yet.

(e) What is happening "in the long run" (as n grows larger without bound)?

12. A gambler from France offers you these two games:

Game 1: You roll a die four times. If you hit a 1 any of the four times, you win.

Game 2: You roll a pair of dice 24 times. If you hit double 1s any of the 24 times, you win.

Aside from the length-of-game issue, which of these games would you rather play to win? Or are they the same?

13. The new toy craze is Mega Men, where kids buy a Mega Man in a box without looking to see which one it is, then open it up when they get home. There are ten toys in all, each equally likely when you buy a box. If you collect all ten, you can make Ultra Mega Mega Man!

On average, how many boxes will you have to buy for your kid before he can finally collect them all?

Sadly, eBay is not an option, since the only cool Mega Men are the ones still in their original packaging.

14. Okay, so there's this game. You get 1 point every time you flip heads. But, anytime you flip tails you're in "danger". If you flip tails a second time *consecutively*, you "bust" and lose all your points (but continue playing).

(a) The game lasts 10 flips. What is the probability that you survive all 10 flips without busting even once?

(b) What is the average score you could expect after 10 flips?

(c) What happens in a longer game? Will the average score increase? Is there a limit?

So, don't flip tails twice in a row. Otherwise it's all good.

Tough Stuff.

15. Do the Yahtzee problem! This shorter version has saved almost 100 pieces of paper, which will all be cut into equal pieces later.

16. Build a data set with at least 5 elements such that if m is the mean and n is the median, then $|m - n|$ is larger than the standard deviation of the set.

Since this is Tough Stuff, we don't have to tell you how to calculate standard deviation.

17. The "Game of the Day" for Tuesday is a shorter description for the actual game. The player has 17 turns to earn 500 points playing the game (1 point times the ball number picked). How would you find the probability that a player would win this game?