2007.13 End Game

Game of the Day: "Press Your Luck"

On "Press Your Luck" players take turns at a board filled with fabulous cash and prizes. But they'll have to avoid the whammy. On the show, the whammy killed off all your money. In this version, the whammy is worth −\$5000. The other possible outcomes are 3000, 6000, 7000, 9000, and 10000, all equally likely. Actually we'll look at the data without the thousands.

Check out these dice! We're really being constructivist now.

Rumor has it that the whammy killed the radio star, too.

1. Find the mean, variance, and standard deviation for one "spin" (or roll). The data set is

$$3, 6, 7, 9, 10, -5$$

2. Find the mean, variance, and standard deviation for two "spins" (or rolls). There are now 36 possible outcomes. Divide the labor if you like.

Union rules prohibit us from telling you how to break up the labor. Intersection rules, too.

3. Fill in the missing pieces here to build an expression you could use to find the distribution for three "spins".

$$(x \quad + x \quad + x^7 + x \quad + x \quad + x \quad)$$

By the way, the mean score for 3 spins is 15 and the variance is 75. (So what's the standard deviation, then?)

4. Use the results from the first three problems to predict the mean, variance, and standard deviation for four spins, 25 spins, 100 spins.

Having to do this 100 times by hand would be a dicey situation.

Important Stuff.

5. (a) You spin the Wheel of Fish once. What is the mean and variance for what comes? (The possible outcomes were 1, 2, 3, 10.)

(b) You roll a standard die once. What is the mean and variance for what comes? (The possible outcomes are 1, 2, 3, 4, 5, 6.)

(c) In the Wheel of Fish bonus round, the player spins the wheel, then also gets to roll a die. They earn the total number of fish from both results. What is the mean and variance for these 24 possible outcomes? The table at the top of the next page might prove useful.

Please, no more fish!!! I thought we were done with the fish!!!

	1	2	3	4	5	6
1						
2						
3						
10						

6. Expand this product:

$$(x + x^2 + x^3 + x^{10})(x + x^2 + x^3 + x^4 + x^5 + x^6)$$

Any thoughts?

Please only include thoughts about this math problem.

7. The actual Wheel of Fish bonus round has the player roll 12 dice at once, then spin the Wheel of Fish six times. The total number earned from all of this is the amount the player will take home, in pounds of fish. Use the properties of mean and variance to find the mean, variance, and standard deviation for how much fish a player stands to win in the bonus round.

¡*No mas pescados*!

8. Flip a coin. If it comes up heads, record a 1. If it comes up tails, record a 0.

(a) If you flip the coin once, what is the mean, variance, and standard deviation for the total count (aka the number of heads)?
(b) *Two* coin flips?
(c) *Three* coin flips?
(d) *Twenty-five* coin flips?
(e) *240* coin flips?
(f) n coin flips?

Hm, 240? That sounds familiar somehow. And yet so long ago.

9. Answer a multiple-choice question with four options without looking. If you were right, record a 1. If you were wrong, record a 0.

(a) If you answer one question, what is the mean, variance, and standard deviation for the total count (the number of questions answered correctly)?
(b) *Two* questions?
(c) *Three* questions?
(d) *Four* questions?
(e) *48* questions?
(f) n questions?

Most of the answers can be found in yesterday's work on the Millionaire game.

Neat Stuff.

10. You run an experiment that has probability p of success. The probability of failure is $1 - p$.

(a) Find the mean and variance for performing the experiment once, in terms of p.

(b) *Two* runs of the experiment?

(c) n runs?

(d) Many statistics books include the rule for standard deviation for "Bernoulli trials" which are experiments with probability of success p and failure q. Explain how you know the rule for the standard deviation σ is

$$\sigma = \sqrt{npq}$$

The last two problems describe two such experiments: rolling a die ($p = .5$) and answering a trivia question ($p = .25$).

Apparently the stat books want you to mind your p's and q's.

11. (a) Find the probability that if you guess at 3 multiple choice questions, you will get exactly 2 of them right. Each question has four possible answers.

(b) Expand this and figure out what is up with that:

$$(0.25r + 0.75w)^3$$

12. Wheel of Fish runs a "Tournament of Champions" where all fish values are doubled: the wheel has 2, 4, 6, and 20. What happens to the mean, variance, and standard deviation for the number of fish in one spin? (They were 4, $\frac{25}{2}$, and $\frac{5}{\sqrt{2}}$ in the original game.)

To qualify for the Tournament of Champions, you must perform the Macarena while eating a fish.

13. Wheel of Fish once ran a Celebrity Week where all fish values were 5 more than normal: the wheel had 6, 7, 8, and 15. What happened to the mean, variance, and standard deviation then?

For the love of Pete, please, please, NO MORE FISH. Celebrity week on "Wheel of Fish" featured Todd Bridges, Florence Stanley, and Abe Vigoda. What, you don't get it?

14. Spin two spinners, and look at the sum of what comes. The first spinner is your generic 1-2-3 spinner with three equal zones. The second spinner has only two zomes, labeled 0 and 3.

(a) Find the mean and standard deviation for the sum you get from the two spinners.

(b) Multiply this out:

$$(x + x^2 + x^3)(1 + x^3)$$

Any thoughts?

As featured in the upcoming Tom Clancy novel "The Sum of All Spinners".

Review Your Stuff.

The final day of this course is mostly taken up by review problems. So we think it would be a good idea for groups to form some summarizing questions that come out of whatever you might find valuable in this course. So, we want your *table* to write *two problems* on any subject that has cropped up in the course.

Here are some topics you might consider writing problems about.

Farey sequences	Common factors or lack thereof
Pascal's triangle	Expected value
Variance	Standard deviation
Counting with polynomials	General probability

The goal is to create a review whose problems get at the fact that we've come a long, long way in three weeks. The problems should help others synthesize their learning of the aforementioned topics.

Ooh, "aforementioned". That's like a three-dollar word.

So, don't write any stumpers; consider yourself writing two problems that could both fit into "Important Stuff." If your table wants to write more than two, that's fine, and the extra questions can be a little more "Neat" or "Tough" or even "Game"-y. We reserve the right to pile your problem(s) into a category called "Random Stuff"; no offense is intended. And it's okay to be funny, as long as it doesn't get in the way of the math.

So if your table wants to submit a game suggestion for the final day, go ahead, but you're still on the hook for two straight-away review problems. No tricks plz!

Tough Stuff.

15. Let $z = x+(\sqrt{1-x^2})i$ be a complex number with $|x| < 1$. Use the nSpire to help here unless you really want some tough algebra.

(a) What's the magnitude of z?
(b) Expand z^2 and look at the real part. Hm? Zeros?
(c) Expand z^3 and look at the real part. Hmmm? Zeros? Connection to some earlier question, perhaps?
(d) Expand z^4 and look at the real part. Hmmmmm?
(e) Graph the real part of z^2, z^3, z^4 as a function of x.
(f) What might z^n look like?

Pafnuty says hi. Who's that? Look it up! And you thought you'd get through the whole course without hearing from him.

Bonus Tracks

This section is reserved for a few things we weren't able to get to in the course. Have a look, play around if you like.

16. It's possible to make two different, distinct dice that *don't* have 1 through 6 on them, but their sums behave the same as the sums from regular dice. The rules for these dice are

- All numbers on each face must be positive integers.
- When the two dice are combined, there is 1 way to get a sum of 2, 2 ways to get a sum of 3, ... 6 ways to get a sum of 7, ... 1 way to get a sum of 12.

What's the only other way to make dice do this? Can factoring help you?

17. Can you make 3 distinct dice that behave like 3 regular dice would? What about 4 dice?

18. Prime numbers seem to get rarer and rarer as the numbers get higher.

(a) How many primes are there between 1 and 100?
(b) ... between 101 and 200?
(c) ... between 201 and 300?
(d) Make a long table of prime numbers, along with the "gap" between them. For example, there is a gap of 6 between 23 and 29.

19. Show that there is no limit to the size of the "gap" between consecutive primes.

20. The gaps between primes seem to not be very well-behaved. But, neither are the primes. One way of trying to smooth this is to look at the *average gap* between primes. For example, the average gap among the first 5 primes is

$$\frac{1+2+2+4}{4} = \frac{9}{4}$$

The first 5 primes are 2, 3, 5, 7, 11; the four gaps are 1, 2, 2, 4.

Calculate the average gap for

(a) the first 10 primes
(b) the first 50 primes (the 50th is 227, starring Jackee)
(c) the first 250 primes (the 250th is 1579)
(d) the first 1250 primes (the 1250th is 10,177)

Any thoughts?

21. The function $p(n)$ takes in n and returns the nth prime.

(a) Show that $\frac{p(n)}{n}$ gives the probability that a positive integer less than or equal to n is prime.
(b) What happens to this function as n gets larger? Does it approach a limit... or...?

22. The function $p(n)$ takes in n and returns the nth prime number. Consider another function that takes in this same n and returns the *average gap* among the first n prime numbers. (You were asked to calculate this function for several values of n.) Get some more data points and investigate this function. What does this investigation suggest about the prime numbers?

23. How does a calculator find the "line of best fit"?

24. Show that the line of best fit always passes through the balance point of a data set; that is, the point $(\overline{x}, \overline{y})$ that would be the center of mass or centroid of the data.

25. Recently we calculated this product:

$$\prod_{p \text{ prime}} \left(1 - \frac{1}{p^2}\right)$$

What happens if you remove the square and try to calculate...

$$\prod_{p \text{ prime}} \left(1 - \frac{1}{p}\right)$$

Use this to prove that there an infinite number of primes.

26. Meanwhile, why stop at squares? Investigate

$$\prod_{p \text{ prime}} \left(1 - \frac{1}{p^s}\right)$$

for various choices of the parameter s. (We called it s for a reason, by the way.)