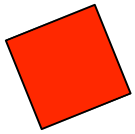
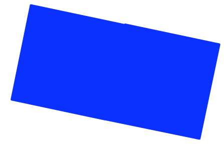


Table 6



Ones and Twos

Using rods of lengths 1 and 2, how many different ways can you make a line of total length n using these rods?

Please explain your findings.

Goals

Students will learn or practice:

- combinatorial math
- algebraic expression
- sequence (Fibonacci numbers)
- pattern recognition
- mathematical modeling
- problem solving
- reasoning
- communication
- representation

Extensions

- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics
- There are many applications using Fibonacci numbers!

Student Responses

Student 1

n	# ways	List (O for rod of length One, and T for rod of length Two)
1	1	O
2	2	T, OO
3	3	TO OT, OOO
4	5	TT, TOO OTO, OOT, OOOO
5	8	TTO, TOT, TOOO OTT, OTOO, OOTO, OOOT, OOOOO
6	13	TTT, TTOO, TOTO, TOOT, TOOOO OTTO, OTOT, OTOOO, OOTT, OOTOO, OOOTO, OOOOT, OOOOOO
…		

I realized that I can form the list of:

n = 3 by attaching "T" to the list of n = 1 and "O" to the list of n = 2. So # ways = 1 + 2 = 3

n = 4 by attaching "T" to the list of n = 2 and "O" to the list of n = 3. So # ways = 2 + 3 = 5

n = 5 by attaching "T" to the list of n = 3 and "O" to the list of n = 4. So # ways = 3 + 5 = 8

n = 6 by attaching "T" to the list of n = 4 and "O" to the list of n = 5. So # ways = 5 + 8 = 13

So, (# ways for n = k) = (# ways for n = k – 2) + (# ways for n = k – 1)

This is a Fibonacci number sequence – $F_n = F_{n-1} + F_{n-2}$

<u>Student 2</u>

n	arrangements (order is not important)	Permutations (order is important)	# ways
1	O	1	1
2	T	1	
	OO	1	1 + 1 = 2
3	TO	$^2C_1 = 2$	
	OOO	1	2 + 1 = 3
4	TT	1	
	TOO	$^3C_1 = 3$	
	OOOO	1	1 + 3 + 1 = 5
5	TTO	$^3C_2 = 3$	
	TOOO	$^4C_1 = 4$	
	OOOOOO	1	3 + 4 + 1 = 8
6	TTT	1	
	TTOO	$^4C_2 / ^2C_1 = 6$	
	TOOOO	$^5C_1 = 5$	
	OOOOOO	1	1 + 6 + 5 + 1 = 13
7	TTTO	$^4C_1 = 4$	
	TTOOO	$^5C_2 / ^2C_1 = 10$	
	TOOOOO	$^6C_1 = 6$	
	OOOOOOO	1	4 + 10 + 6 + 1 = 21
…			

I can see that

(# ways for n = 3) = (# ways for n = 1) + (# ways for n = 2)
(# ways for n = 4) = (# ways for n = 2) + (# ways for n = 3)
(# ways for n = 5) = (# ways for n = 3) + (# ways for n = 4)
(# ways for n = 6) = (# ways for n = 4) + (# ways for n = 5)
(# ways for n = 7) = (# ways for n = 5) + (# ways for n = 6)

I know this is a Fibonacci sequence, but I cannot explain why…

Student 3

Let x be the number of rods of length 1.
Let y be the number of rods of length 2.

$1x + 2y = n$

Since there are 2 variables and 1 equation, this means there may be multiple solutions.

n	x (length 1)	y (length 2)	# ways
1	1	0	1
2	2	0	
	0	1	2
3	1	1	
	3	0	2
4	0	2	
	2	1	
	4	0	3
5	1	2	
	3	1	
	5	0	3
6	0	3	
	2	2	
	4	1	
	6	0	4
7	1	3	
	3	2	
	5	1	
	7	0	4
…			

ways = 1 for n = 1

ways = 2 for n = 2, 3

ways = 3 for n = 4, 5

ways = 4 for n = 6, 7

of ways = integer part of ((n + 1) / 2)

This solution is incorrect because the student forgot to account for the different arrangements of each combination of x and y.

Nevertheless, the student derived a formula for the answers he/she got.

Student 4

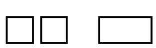
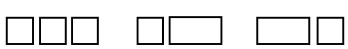
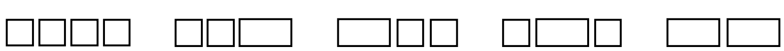
Algebra for grades 9-12
You are given rods of lengths 1 and 2.

How many different ways can you make a line of total length *n* using these rods?

	Sum of 1	Sum of 2	Sum of 3	Sum of4	Sum of 5	Sum of 6
	1	1+1	1+2	2+2	2+2+1	
		2	2+1	2+1+1	2+1+2	
			1+1+1	1+2+1	1+2+2	
				1+1+2	1+1+1+2	
				1+1+1+1	1+1+2+1	
					1+2+1+1	
					2+1+1+1	
					1+1+1+1	
# of Combinations	1	2	3	5	8	13

Predict sum of 6 will have 13 combinations with the "1" and "2" rods. I noticed that if I add the two previous combinations, they add up to the next one.

Student 5

length		ways
length 1		1
length 2		2
length 3		3
length 4		5
length 5		8?

I predict that with 6 there would be 13 ways because you add the two previous numbers.

Algebra for grades 9-12

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 9–12 all students should—
Understand patterns, relations, and functions	• generalize patterns using explicitly defined and recursively defined functions; understand relations and functions and select, convert flexibly among, and use various representations for them;
Represent and analyze mathematical situations and structures using algebraic symbols	• use symbolic algebra to represent and explain mathematical relationships; • use a variety of symbolic representations, including recursive and parametric equations, for functions and relations; • judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
Use mathematical models to represent and understand quantitative relationships	• identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships; • use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts; • draw reasonable conclusions about a situation being modeled.

NCTM STANDARDS

Problem Solving Standard for Grades 9–12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving

Communication Standard for Grades 9–12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Connections Standard for Grades 9–12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Representation Standard for Grades 9–12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena