

Towers of Hanoi

The Game: A number of disks of different sizes are stacked on a post. Each disk is smaller than the one below it. There are two adjacent posts, and the object is to move all of the disks to either of the other posts. The disks can only be moved one at a time, and no disk may be placed on top of a smaller one.

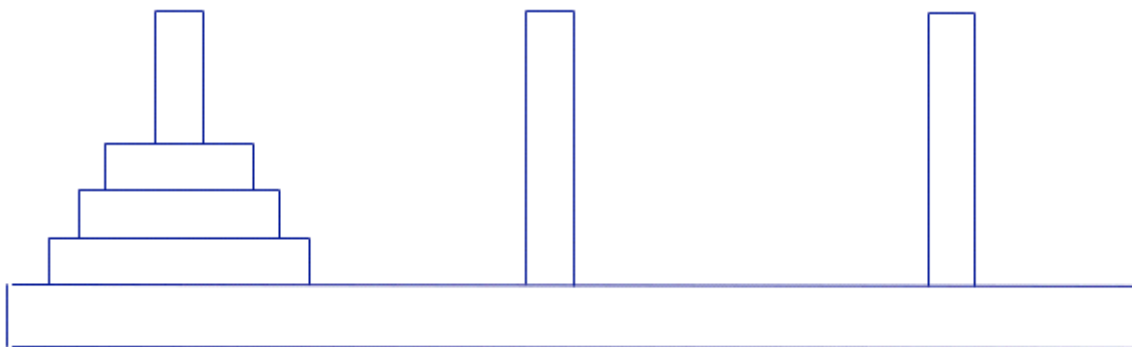
Problem Statement:

How many moves does it take to move 25 disks?

Extension:

Legend has it that in Hanoi, there is a Buddhist temple with a version of the game that has 64 gold disks. The monks work on moving the disks 24 hours a day, and they move one disk every minute. When they finish moving the disks from the first post to the third, the world will come to an end. They have been at it for a thousand years. According to this legend, when will the world end?

The formula $A_n = 2^n - 1$ appears in other real life situations. For example, in a single elimination tournament (such as NCAA basketball), the number of games played is $2^n - 1$ where n is the number of rounds played. How many games would be played if there were 128 teams.



Student Response #1

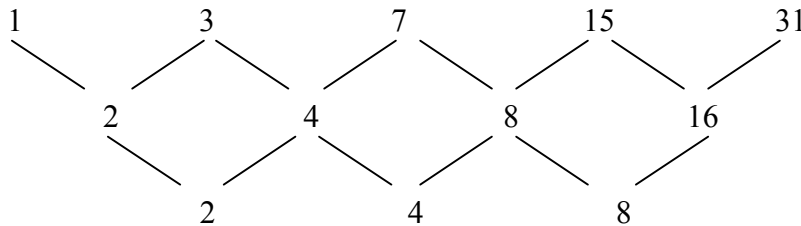
The most likely method of solving starts by finding out how many moves are needed for small numbers of disks and looking for a pattern. Using paper disks on a marked piece of paper, students can find out how many moves it takes to move 1 to 5 disks.

# of disks	1	2	3	4	5
# of moves	1	3	7	15	31

Students may observe that, in order to move a disk, it is first necessary to move all of the disks above it, then move the disk, and then move the others back on top of it. To move 4 disks, first you have 7 moves to get the top three off of the bottom disk, 1 move to move the fourth disk, and then 7 more to get the first three back on top. From this, the student may come up with an equation on the order of $A_n = 2A_{n-1} + 1$.

Student Response #2

Students may investigate relationships by looking at differences



and notice that the numbers increase by a constant ratio. This may lead to the equation $A_n = 2^n - 1$.

Student Response #3

Other, although less productive answers might be attempted by finding the number of moves for 5 (31) and multiply it by 5 or square the number in the mistaken assumption that one of these processes will give the same result as continuing the process.

Activity Appropriateness

Mathematical Content:

- Exponential increase
- Pattern recognition
- Regression
- Recursion

Audience:

- Algebra 1

Numerous extensions are possible to explore and encourage further mathematical development.

Purpose

Students will investigate patterns and use their findings to develop a formula to predict the solution for any given number of iterations.

Process

Students will play the game “Towers of Hanoi”. Students will be given a description of the rules of the game. As students play the game, their job will be to investigate the number of moves it takes to move 1 disk, 2 disk, 3 disk, 4 disk, 5 disk. The students will look at their findings and recognize patterns. They should be able to formulate an equation that would allow them to solve for the minimum number of moves for any number of disks.