5 *Nelly (f. Tim McGraw)*

PROBLEM

Calculate this to seven decimal places:

$$\frac{1}{\sqrt{5}} \cdot \left(\frac{1+\sqrt{5}}{2}\right)^9$$

What's up with that?

Useful Stuff.

0. What's the Quadratic Formula?

The #1 answer: "The bane of my Algebra I existence."

1. Find a closed form for the recursive rule

$$t_n - 8t_{n-1} + 15t_{n-2} = 0$$

with the starting values $(5, 19)$.

2. Write a recursive rule that matches each of these closed-form rules:

(a) $t_n = 3 \cdot 5^n + 5 \cdot 4^n$
(b) $t_n = 11 \cdot 5^n - 2 \cdot 4^n$
(c) $t_n = A \cdot 5^n + B \cdot 4^n$

Send a message in a bottle to the language police on this poor use of grammar. Or *is* it...!

3. A geometric sequence starts $1, r, \ldots$.

(a) What is the next term?
(b) Two geometric sequences that start $1, r, \ldots$ satisfy the recursive rule

Eh, no rule, guess you're done.

$$t_n = 10t_{n-1} - 24t_{n-2}$$

Guess you're not done.

What are they?

(c) What's the general rule for *any* sequence that satisfies the recursive rule $t_n = 10t_{n-1} - 24t_{n-2}$?

4. So what's going on here? What relationship is there between the numbers in the recursive rule and the closed form?

5. Two geometric sequences that start $1, r, \ldots$ satisfy the recursive rule

$$t_n = 2t_{n-1} + t_{n-2}$$

What are they?

6. Find a closed-form rule that fits this sequence:

$$2, 2, 14, 38, 146, 482, \ldots$$

You'll probably have to figure out the recursive rule first!

Finding the closed form on another table's scrap paper does *not* count.

7. Find a closed-form rule that fits this sequence:

$$0, \sqrt{5}, \sqrt{5}, 2\sqrt{5}, 3\sqrt{5}, 5\sqrt{5}, 8\sqrt{5}, 13\sqrt{5}, \ldots$$

8. Find a closed-form rule for the Lucas numbers:

$$2, 1, 3, 4, 7, 11, 18, 29, \ldots$$

These numbers come from a galaxy far, far away... or maybe they're just numbers Hava wrote on the board in Cinti.

9. *Upshot alert!* Find a closed-form rule for the Fibonacci numbers:

$$0, 1, 1, 2, 3, 5, 8, 13, \ldots$$

10. (a) Say, what's the average of the two sequences in problems 7 and 8?

(b) How cool is that?

Neat Stuff.

Today's Neat Stuff features 100% non-recycled material.

11. Find a closed-form rule that fits this sequence:

$$2, 2+i, 3, 8-i, 17, 32+i, 63, 128-i, 257, \ldots$$

The recursive rule is actually kind of a pain here. Check out the numbers and see if you can find the closed-form rule. What's this i thing?

12. Find a recursive rule that fits this sequence:

$$1, 3, 4, 2, -4, -12, -16, -8, 16 \ldots$$

13. Find a closed-form rule that fits the sequence in problem 12. *Uh-oh!* See "Tough Stuff" for more.

14. (a) Toppum makes all 8 trains of length 4. How many whites did he use?

(b) Same question, but about the 16 trains of length 5.

(c) Complete this table for the *total number of whites* used in making all trains of length n:

1	2	3	4	5	6	7	8
1	2						

(d) Alright, what's the recursive rule here? Surely you didn't actually make all 128 trains of length 8.

(e) Find a closed-form rule, now that you know how. Warning: messy radical arithmetic ahead!

We named the first one Thomas, and the eighth one Percy.

When Castro counts up all his cigars... that's radical arithmetic!

15. Find this sum exactly:

$$0 + \frac{1}{10} + \frac{1}{100} + \frac{2}{1000} + \frac{3}{10000} + \cdots + \frac{F_n}{10^n} + \cdots$$

Here, F_n is the nth Fibonacci number, and the sum continues forever.

Tough Stuff.

16. Use a picture of the complex plane to explain just what the heck is going on in problem 13.

17. Here's a sequence that comes from a recursive rule:

$$1, 6, 45, 216, 891, \ldots$$

Find the recursive rule and the closed-form rule. Isn't that interesting?

18. Find this sum exactly:

$$0 + \frac{1}{100} + \frac{4}{10000} + \frac{9}{1000000} + \cdots + \frac{n^2}{10^n} + \cdots$$

As a decimal, this is $0.0104091625\ldots$. Everything gets all jumbly after $n = 10$, but the sequence rule still works.