

Teacher Notes

Our Backpack Data: Making Conclusions from Graphs

Abstract:

In this group activity, students are asked to match different statistical questions from the PCMI backpack data to an appropriate visual display. Students are divided into groups, and given 5 questions about the study. They also receive eight visual displays generated from the data collected. In small groups, students choose graphs, and make well-justified conclusions about each question. Students are expected to base conclusions on specific evidence from chosen graph(s). Groups join together to compare the different conclusions found from each group, and discuss how different graphs may be used to answer the same question. In this discussion period, they ask questions that require classmates to communicate reasoning with detail and clarity. Finally, students create a new question, choose (or create) two graphs to answer the question, and make conclusions based on the graphs.

Procedure: In this group activity, students are asked to match up different statistical questions from the PCMI backpack data to an appropriate visual display. Students should be familiar with the data collection process for the backpack data: 51 adults participating in the 2006 Park City Mathematics Institute were asked to estimate the weight of their backpacks (book bags) in pounds. Each subject's backpack was then weighed with a typical bathroom scale. The guessed and actual weights were recorded in pounds. The data file **backpack_data_2006.ftm** contains guessed and actual weights, as well as the gender of each subject, and the difference between their guess and the actual weight (guess-actual). The instructor may choose to have students collect data in a similar fashion, and replace the given data.

For Parts I and II, students should be divided into groups of two or three. Each student should be given the handout above, with 8-10 copies of the response template.

Part I: In small groups, students should choose at least one graph that can answer each question. However, some questions can be answered by more than one graph. Students should be prepared not only to choose graphs, but also to prepare an answer for each question and justify any conclusions with specific evidence from the chosen graph(s). Each small group should complete one response template for each answer provides. Some small groups may elect to answer a question in more than one way, but should be encouraged to provide at least one complete response for each question.

Part II: After small group discussion, each group should be invited to present results by attaching each response with corresponding graph(s). The instructor should go through each statistical question one at a time, and invite 2-3 different groups to describe and explain choices. The whole class can then compare different conclusions found from each group, and discuss the explanations. Because some of the questions are broad enough to invite interpretation, the instructor should look for opportunities to highlight

different interpretations of the same question. Students should be encouraged to ask questions that require classmates to communicate reasoning with detail and clarity.

Part III (Extensions): If students have access to Fathom or another data analysis program, Part III can be an open-ended take-home assignment that should take about 30-45 minutes. Students should be provided with the raw data, and create a new question, choose (or create) two graphs to answer the created question, and make conclusions based on graphs.

Analyzing Data: The table below can be used to determine how to use each graph to answer the question. If the graph is not usable, the box is blank.

	<i>How Heavy</i>	<i>Better guessers: M or F?</i>	<i>What fraction missed by less than 3 lbs?</i>	<i>Are people good at guessing?</i> Note: "Good" is a term that needs to be defined by students. Answers will vary.	<i>Overestimate/Underestimate?</i>
Graph A		Women. A majority of women are under 3 lbs, but a majority of men are 3 lbs or more.	26/52 = 50% missed by 3 lbs or under.	If we accept "under 3 lbs" as "good," then about half are "good."	
Graph B	Combine frequencies for M and F, to notice a center weight near 11 lbs. Most individuals are within 4-5 pounds. There is some right skewness. Students may also notice that men carry heavier backpacks.				
Graph C		Women. 13/16 of women are within 5 lbs, but a much smaller proportion of male dots are within this range.	By marking off -3 and +3 on the "differences" axes, about half of the points are within these bounds.	This graph is most useful. Students can define good in many ways and use this graph	The number of individuals with difference > 0 is slightly higher than the number < 0. Perhaps a slight tendency to overestimate, but unclear if true for population.
Graph D	The x-coordinates of most of the points on the scatter plot show "actual" weight near 10, give/take 4-5 pounds. A few are scattered considerably higher-- suggesting actual weights are skewed right slightly.		Draw the lines $y = x + 3$, $y = x - 3$; observe about half the points within these two boundaries.	This graph is also useful. Students can define good in many ways and use this graph to determine % points sufficiently close to "actual = guessed."	The number of points above "guessed = actual" is slightly higher than the number below. Perhaps a slight tendency to overestimate, but unclear if true for population.
Graph E		The IQR for women is much smaller than for men.			The medians for each box plot are both near zero, with the men slightly skewed right. Perhaps a slight tendency to overestimate for men, hard to tell for entire population.
Graph F	"Actual" box plot shows median near 10 lbs. slightly skewed right in the top quartile, but 1/4 of all between 10 and 13 lbs.				
Graph G		For women 11/16 under 3 lbs, a much higher proportion than 15/36 for men in the study.	26/50 dots are in the under 3 lbs category.	If we accept "under 3 lbs" as "good," then about half are "good."	
Graph H				Suppose "good" means within 2 lbs. Then find percentile for difference = 2, and difference = -2. ; subtract. That proportion is "good."	50 th %ile at 0, but 20 th at -2, and 80 th at +4. Slight skew right in differences. Suggest when incorrect, overestimates are larger than underestimates.

Part II: More Issues for discussion:

- a. **Sureness of Conclusions:** When students make conclusions, how confident are they about their answers? What facts would allow them to be more confident about their conclusions? More data? More information about the data collection process? More clear definitions?
- b. **Percentile Plot:** Although students may be familiar with percentiles, they may have never seen a percentile plot. These are also called *cumulative frequency plots*, or *ogives*. It is valuable to monitor group discussions so that all students understand how to read these graphs. It is also useful to have students construct a box plot from the percentile plot as a way to assess understanding.
- c. **Data Collection** The data from adults was collected between 7:30 a.m. and 8:00 a.m. before morning sessions at a mathematics conference. Do these facts have an influence?
- d. **Measuring percentage errors:** In defining a “good” guess, some students may want to define “error” in terms of a percentage of the actual weights. This idea is indeed an effective way of measuring errors. Only the scatter plot can allow us to measure an individual’s percentage error. For example, if a student wants to know which points fall 20% above or 20% below the actual weight, the student should graph the lines “guess” = 1.2 “actual” and “guess” = 0.8 actual, respectively.

Part III: Extensions

- a. The first extension can be used as an end-of-topic assessment. A possible rubric for assessment follows:

Criterion	Grade			Comments
	0	1	2	
Questions: the 2 or 3 questions asked are clearly defined.				
Graph chosen is appropriate for the question asked.				
Overall Conclusion: Makes appropriate conclusions without over-reaching.				
Use of evidence: Student points out specific evidence in justifying the conclusion.				
Communication is clear and concise. Student's use of statistical terms is generally correct, and appropriate for what has been learned.				

2 pts. strong mastery, free of errors.

1 pt. partial mastery, slight errors.

0 pts. non-mastery: major errors.

- b. Students may collect their own backpack data, and make comparisons about guessing against adults in the data. Other variables to consider that may be related to backpack weights and guesses are:

- Whether the students lifts weights
- Grade level
- Whether the backpack is light or heavy (may make it easier to estimate weight)
- Time of day (maybe backpacks are heaviest in the morning, but fewer books are carried during lunchtime)