SECOND ANNUAL DREXEL MATH CONTEST

Directions: No calculators are permitted. Use the blank paper provided to solve the problems. Put your name on every page, and clearly label the problem you are working on. For each problem, check your work and then enter your final answer here on the answer page.

Name:
Grade:
High School:
Name of the teacher here with you:

Your Answers:

[1] _____ **[8]** _____

[2] _____ **[9]** _____

[3] _____ **[10]** _____

[4] _____ **[11]** _____

[5] _____ **[12]** _____

[6] _____ **[13]** _____

[7] _____ **[14]** _____

1. Find two positive integers a and b such that the following three conditions are all satisfied:

   - the greatest common divisor of a and b is 15, and
   - the least common multiple of a and b is 225, and
   - $5b = 3a$.

   Answer: $a = 75, b = 45$.

2. What integer is equal to $\frac{\frac{2}{11}}{\frac{8}{11}-\frac{5}{7}}$?

   Answer: 14.

3. Tortoise and Hare had a 10 mile race. Hare sprinted at a pace of 10 miles per hour for 5 minutes. Then the cheating Hare grabbed a bicycle, and pedaled off at 20 m.p.h. for the next 25 minutes. The arrogant Hare then stopped, lay down, and fell asleep for one hour and twenty five minutes. Finally, Hare woke up and sprinted at 10 miles per hour for the rest of the race. Meanwhile, the honest Tortoise had jogged at a steady pace for the entire race. If the race was a tie, how fast did Tortoise jog?

   Answer: 5 m.p.h.

4. Find the area of the quadrilateral with vertices at $A = (0, 0)$, $B = (4, 4)$, $C = (8, 6)$, and $D = (10, 2)$.

   Answer: 26

5. Find the real number x such that $\left(\sqrt[5]{\frac{x}{2}} + 6\right)^{1/3} = 2$.

   Answer: $x = 64$.

6. What is the remainder when $x^5 + 2x + 1$ is divided by $x^3 - x - 1$?

   Answer: $x^2 + 3x + 2$.

7. What is the remainder when $x^{2003} + x^{16} + 12$ is divided by $x^2 + 1$?

   Answer: $13 - x$.

8. The square in Figure 1 has sides of length 1. Assume that each of the Q_i's is one third of the way along its edge, e.g. $|\overline{P_1Q_1}| = \frac{1}{3}$. As shown, R_1 is the point where $\overline{P_1Q_2}$ intersects $\overline{P_4Q_1}$, and similarly for R_2, R_3 and R_4. Find the area of the quadrilateral $R_1R_2R_3R_4$.

   Answer: $\frac{2}{5}$

9. For what integer x is $\frac{1}{\log_2 x} + \frac{1}{\log_5 x} = \frac{1}{100}$.

   Answer: $x = 10^{100}$ (one googol).

10. Which real numbers x are in the domain of $f(x) = \log_3 \log_3 \log_3(x + \frac{2}{x})$?

    Answer: $(0, 1) \bigcup (2, \infty) = \{x |\ x < 1 \text{ or } x > 2\}$

11. Find the area of the region bounded by the curve $x^2+y^2-10x-10|y|=0$.

    Answer: $50+75\pi$.

12. The disk $x^2-6x+y^2-20y+104\leq 0$ is partitioned into three pieces by the lines $y=2x+4$ and $y=-3x+14$. Find the areas of the three pieces.

    Answer: $\frac{5\pi}{2}$, $\frac{5\pi-10}{4}$, and $\frac{10+5\pi}{4}$

13. Suppose that ΔABC is a triangle with $|\overline{AB}|=7, |\overline{AC}|=5$, and $|\overline{BC}|=6$. Suppose that D is point on $\overline{BC}$ such that $\angle BAD = \angle CAD$, i.e. $\overline{AD}$ bisects the angle at A. Find $|\overline{BD}|$.

    Answer: $|BD|=7/2$.

14. Find the equation of the parabola that passes through the points $(0,5),(1,3)$ and $(2,5)$, and has an axis of symmetry parallel to the y-axis.

    Answer: $y=2x^2-4x+5$.